Semi-annual Workshop in Dynamical Systems and Related Topics
Pennsylvania State University, Sept. 26 - 29, 2019

Organizers: Svetlana Katok and Yakov Pesin

Schedule of talks

THURSDAY, September 26

1:00 - 1:50 Registration
1:50 - 2:00 Opening Remarks
2:00 - 2:50 Raphaël Krikorian
On the divergence of Birkhoff Normal Forms
3:00 - 3:30 Departmental Tea
3:35 - 4:25 Department of Mathematics Colloquium
Federico Rodriguez Hertz
Rigidity in hyperbolic dynamics
4:40 - 5:30 Sebastien Gouëzel
Ruelle resonances for linear pseudo-Anosov maps

FRIDAY, September 27

9:00 - 9:50 Keith Burns
Openness of accessibility for partially hyperbolic diffeomorphisms with three dimensional center
10:00 - 10:30 Coffee break and Poster session
10:30 - 11:20 Davi Obata
Stable ergodicity for certain partially hyperbolic systems with mixed behavior along the center
11:30 - 12:20 Clark Butler
Global Lyapunov spectrum rigidity for geodesic flows of negatively curved symmetric spaces
12:30 - 2:00 Lunch break

Special Session on Katok-Spatzier Global Rigidity Conjecture

2:00 - 2:50 Ralf Spatzier
On global rigidity of higher rank commuting actions - An Overview
3:00 - 3:50 Kurt Vinhage
Classification of Cartan actions of abelian groups
4:00 - 4:30 Coffee break
4:30 - 5:20 Danijela Damjanovic
What we learned from Katok-Spatzier conjecture
5:30 - 6:20 Problem Session

7:00 - Conference Banquet at The Gardens Restaurant
Penn Stater Hotel & Conference Center,
215 Innovation Blvd State College, PA 16803
SATURDAY, September 28

9:00 - 9:50  Manfred Denker
            *An ergodic theorem for multivariate functions*

10:00 - 10:30 Coffee break and Poster session

**Short talk sessions:**

10:30 - 10:55  Kitty Yang (113)
                *Mapping class group of minimal subshifts*

10:30 - 10:55  Ilya Gekhtman (114)
                *Harmonic measures for random walks vs Gibbs states for geodesic flows for negatively curved geometrically finite manifolds*

11:00 - 11:25  Diaaeldin Taha (113)
                *On cross-sections to the horocycle & geodesic flows on quotients of SL(2, R) by Hecke groups*

11:00 - 11:25  Salman Siddiqi (114)
                *Exponential mixing for certain isometric extensions of Anosov flows*

11:30 - 11:55  Minsung Kim (113)
                *Effective equidistributions of higher step nilflows*

11:30 - 11:55  John Alexander Arredondo (114)
                *On infinitely generated Fuchsian groups of some infinite genus surfaces*

12:00 - 12:50  Zhenqi Wang
                *New progress of local rigidity of exceptional algebraic actions*

1:00 - 2:30  Lunch break

2:30 - 3:20  Mark Pollicott
            *Average word length for closed geodesics on surfaces*

**Brin Prize Session:**

3:30 - 3:50  Prize Ceremony

4:00 - 4:30  Coffee break

4:30 - 5:20  Talk 1

5:30 - 6:20  Talk 2
SUNDAY, September 29

9:00 - 9:50 Rodrigo Treviño
*What we talk about when we talk about renormalization*

10:00 - 10:30 Coffee break

**Short talk sessions:**

10:30 - 10:55 Yaofeng Su (113)
*Random dynamical system and quenched invariance principle*
Emily Smith (114)
*Unique ergodicity for special classes of measured foliations*

11:00 - 11:25 Jing Zhou (113)
*A rectangle billiard with moving slits*
Aaron Benda (114)
*Weakly mixing systems with dense prime orbits*

11:30 - 12:20 Vaughn Climenhaga
*Counting closed geodesics on surfaces without conjugate points*

12:30 - 1:20 Giovanni Forni
*Countable Lebesgue spectrum for area-preserving toral flows*
**Titles and Abstracts**

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**John Alexander Arredondo** (Fundacion Universitaria Konrad Lorenz) *On infinitely generated Fuchsian groups of some infinite genus surfaces.* In this talk, for a non-compact Riemann surface $S$ homeomorphic to either: the Infinite Loch Ness monster, the Cantor tree and the Blooming Cantor tree, we will present a precise description of an infinite set of generators of a Fuchsian group $\Gamma < PSL(2, \mathbb{R})$, such that the quotient space $\mathbb{H}/\Gamma$ is a hyperbolic Riemann surface homeomorphic to $S$. For each one of these constructions, we will exhibit a hyperbolic polygon with an infinite number of sides and give a collection of Möbius transformations identifying the sides in pairs.

**Aaron Benda** (University of Maryland, College Park) *Weakly mixing systems with dense prime orbits.* We study orbits sampled along the prime numbers for some weakly mixing, analytic flows on the two-torus. We show that (under certain conditions on these flows) every such orbit is equidistributed along a subsequence. As a consequence, every point’s orbit along primes is dense.

**Keith Burns** (Northwestern University) *Openness of accessibility for partially hyperbolic diffeomorphisms with three dimensional center.* Consider partially hyperbolic diffeomorphisms with an invariant splitting into three bundles: unstable, center, and stable. Didier showed that if the center bundle is one dimensional, then the property of accessibility is open, i.e. it persists under small perturbations of the diffeomorphism. More recently Avila and Viana showed that same is true when the center bundle is two dimensional. The talk will describe an effort to adapt Avila and Viana’s technique to show that accessibility is still open when the center bundle is three dimensional. This is joint work with Jana Rodriguez Hertz and Raul Ures.

**Clark Butler** (Institute for Advanced Study) *Global Lyapunov spectrum rigidity for geodesic flows of negatively curved symmetric spaces.* We show that a smooth Anosov flow which is orbit equivalent to the geodesic flow of a closed negatively curved locally symmetric space $X$ of dimension at least three is smoothly orbit equivalent to that geodesic flow if the Lyapunov spectra of these flows around all periodic orbits coincide up to a scalar multiple. In the special case that this Anosov flow is the geodesic flow of another closed negatively curved manifold $Y$, we conclude from this that $Y$ is homothetic to $X$. We deduce the Mostow rigidity theorem as a corollary. The proof mixes techniques from hyperbolic dynamics and quasiconformal mapping theory in the spirit of Mostow’s proof. We will also pose a “uniformization conjecture” for closed negatively curved manifolds $Y$ whose geodesic flows have periodic Lyapunov spectra matching those of a locally symmetric space, together with some evidence toward this conjecture.

**Vaughn Climenhaga** (University of Houston) *Counting closed geodesics on surfaces without conjugate points.* For negatively curved Riemannian manifolds, Margulis gave an asymptotic formula for the number of closed geodesics with length below a given threshold. I will describe joint work with Gerhard Knieper and Khadim War in which we obtain the corresponding result for surfaces without conjugate points by first proving uniqueness of the measure of maximal entropy and then following the approach of recent work by Russell Ricks, who established the asymptotic estimates in the setting of CAT(0) geodesic flows.

**Danijela Damjanovic** (Kungliga Tekniska Hogskolan, Stockholm) *What we learned from Katok-Spatzier conjecture.* Roughly 25 years of efforts towards proving the Katok-Spatzier conjecture led to development of valuable tools and techniques, and improved vastly our understanding of hyperbolic dynamics. Partially...
hyperbolic actions are much less understood, however understanding of higher-rank Anosov actions can be used to some extent to obtain classification for certain classes of partially hyperbolic actions. In this talk I will discuss several recent results for partially hyperbolic actions, which draw inspiration from, or are consequences of the conjecture and techniques which were developed along the way. This is joint work with D. Xu and A. Wilkinson.

**Manfred Denker** (University of Göttingen) *An ergodic theorem for multivariate functions.* For a measure preserving transformation $T$ of a probability space $(X, \mathcal{F}, \mu)$ and some $d \geq 1$ we investigate sums of the form

$$x \rightarrow \frac{1}{C_n} \sum_{0 \leq i_1, \ldots, i_d < n} f(T^{i_1}(x), \ldots, T^{i_d}(x)), \ n = 1, 2, \ldots,$$

where $C_1, C_2, \ldots$ are normalizing constants and the kernel $f$ belongs to an appropriate subspace in some $L_p(X^d, \mathcal{F}^\otimes d, \mu^d)$. Several applications will also be provided.

**Giovanni Forni** (University of Maryland, College Park) *Countable Lebesgue spectrum for area-preserving toral flows.* In joint work with B. Fayad and A. Kanigowski, we study the spectral measures of conservative mixing flows on the 2-torus having one degenerate singularity. We show that, for a sufficiently strong singularity, the spectrum of these flows is typically Lebesgue with infinite multiplicity. For this, we use two main ingredients: 1) a proof of absolute continuity of the maximal spectral type for this class of non-uniformly stretching flows that have an irregular decay of correlations, 2) a geometric criterion that yields infinite Lebesgue multiplicity of the spectrum and that is well adapted to rapidly mixing flows. This is the first result establishing countable Lebesgue spectrum for non-homogenous zero-entropy dynamical systems. In a joint work with Ulcigrai we proved a few years ago that smooth-time changes of horocycle flows have Lebesgue maximal spectral type. Our criterion applies to this case as well, thereby proving countable Lebesgue spectrum in this case, as conjectured by A. Katok and J. P. Thouvenot.

**Ilya Gekhtman** (University of Toronto) *Harmonic measures for random walks vs Gibbs states for geodesic flows for negatively curved geometrically finite manifolds.* We compare and contrast two classes of measures on the boundary of the universal cover of a geometrically finite negatively curved manifold: stationary (or harmonic) measures for random walks and conditionals for equilibrium states (or Gibbs measures) associated to Hölder potentials. We show that both of these measures can be associated to geodesic flow invariant measures on the unit tangent bundle, with respect to which closed geodesics satisfy different equidistribution properties. We also show that the equivalence of a stationary measure and an equilibrium state can be rephrased in terms of a certain quantitative criterion (relating entropy with generalizations of drift and volume growth), and that when the manifold is not convex cocompact, the two measures are always singular. Most of this is based on separate joint works with Gerasimov-Potyagailo-Yang and with Tiozzo.

**Sebastien Gouëzel** (CNRS & Nantes University) *Ruelle resonances for linear pseudo-Anosov maps.* The Ruelle resonances of a dynamical system are spectral characteristics of the system, describing the precise asymptotics of correlations. While their existence can often be shown by abstract spectral analysis arguments, it is in general not possible to compute them exactly. After explaining the general background, illustrated by classical examples, I will focus on a specific example: in the case of linear pseudo-Anosov maps, Ruelle resonances can be completely described in terms of the action of the map on cohomology. Joint work with Faure and Lanneau.
Minsung Kim (University of Maryland, College Park) **Effective equidistributions of higher step nilflows.** The main result of this presentation is to prove upper bounds for ergodic averages for nilflows on general higher step nilmanifolds. Giovanni Forni and Livio Flaminio established estimates on the speed of equidistribution of nilflows on higher step Quasi-abelian (or Filiform) nilmanifolds. This work shows that the speed of ergodic average of nilflows with Diophantine conditions is polynomial for almost all points. In this talk, we will introduce the special class of nilpotent Lie algebras satisfying the condition called “transversality”, and we will also present the speed of effective equidistribution of nilflows that is determined by the step size and the number of elements in Lie algebras.

Raphaël Krikorian (Université de Cergy-Pontoise) **On the divergence of Birkhoff Normal Forms.** A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent. This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

Davi Obata (Université Paris-Sud) **Stable ergodicity for certain partially hyperbolic systems with mixed behavior along the center.** In the last three decades, several works have been done about stable ergodicity. Due to the famous Pugh-Shub conjecture, most works were done in the partially hyperbolic scenario. In this scenario one of the key properties used is accessibility (or essential accessibility). In this talk I will present the proof of stable ergodicity of a partially hyperbolic skew product introduced by Pierre Berger and Pablo Carrasco. It has two-dimensional center, with expansion/contraction along center and it does not admit any further dominated splitting of the center, and it is non-uniformly hyperbolic. This proof does not use accessibility.

Mark Pollicott (Warwick University & Aix Provence) **Average word length for closed geodesics on surfaces.** For each closed geodesic on a compact negatively curved surface we can associate two values: the length (with respect to the Riemannian metric) and the word length (with respect to an appropriate choice of generators of the fundamental group). We shall consider how the average of the word lengths of closed geodesics with length at most $T$, say, behaves as $T$ tends to infinity.

Federico Rodriguez Hertz (Penn State University) **Rigidity in hyperbolic dynamics.** In this talk I will overview some old and new results in rigidity theory of hyperbolic systems. Two basic questions in this setting are: what makes a hyperbolic system smoothly isomorphic to an algebraic system and when are two systems smoothly isomorphic. I will try to give a glimpse of the problems to address these questions and the results.
Salman Siddiqi (University of Michigan, Ann Arbor) *Exponential mixing for certain isometric extensions of Anosov flows.* I will provide some context, and explain how to obtain exponential decay of correlations for certain (locally accessible) isometric extensions of Anosov flows, using symbolic transfer operators.

Emily Smith (University of Utah) *Unique ergodicity for special classes of measured foliations.* The orbits of the geodesic flow on a surface with a flat metric determine a dynamical system called a measured foliation. Masur and Veech independently proved that a typical foliation is uniquely ergodic: there is only one invariant measure, which is ergodic. However, for a specific measured foliation it is often nontrivial to prove unique ergodicity. We will discuss some new techniques for proving unique ergodicity, in particular for foliations on genus 1 and 2 surfaces. We will use this result to construct some new examples of uniquely ergodic measured foliations on genus 2 surfaces with unusual properties.

Ralf Spatzier (University of Michigan, Ann Arbor) *On global rigidity of higher rank commuting actions - An Overview.* I will discuss the developing theory of commuting abelian actions and their rigidity properties, emphasizing recent results. This talk will serve as an introduction to the Special Session on the Katok-Spatzier Global Rigidity Conjecture.

Yaofeng Su (University of Houston) *Random dynamical system and quenched invariance principle.* We obtain a quenched almost sure invariance principle (QASIP) for random dynamical systems. Applications include random dynamical systems with uniform spectral gap, random Liverani-Saussol-Vaienti maps with indifferent fixed point, random Young Towers. We will also show that many i.i.d. perturbations of nonuniformly expanding maps can be described by a random Young Tower, so QASIP holds for such random systems.

Diaaeldin Taha (University of Washington) *On cross-sections to the horocycle and geodesic flows on quotients of SL(2,\mathbb{R}) by Hecke groups.* In this talk, we explore explicit cross-sections to the horocycle and geodesic flows on SL(2,\mathbb{R})/G_q, with q \geq 3. Our approach relies on extending properties of the primitive integers \mathbb{Z}^2_{prim} := \{(a, b) \in \mathbb{Z}^2 | \gcd(a, b) = 1\} to the discrete orbits \Lambda_q := G_q(1, 0)^T of the linear action of G_q on the plane \mathbb{R}^2. We present an algorithm for generating the elements of \Lambda_q that extends the classical Stern-Brocot process, and from that derive another algorithm for generating the elements of \Lambda_q in planar strips in increasing order of slope. We parametrize those two algorithms using what we refer to as the symmetric G_q-Farey map, and G_q-BCZ map, and demonstrate that they are the first return maps of the geodesic and horocycle flows resp. on SL(2,\mathbb{R})/G_q to particular cross-sections. Using homogeneous dynamics, we then show how to extend several classical results on the statistics of the Farey fractions, and the symbolic dynamics of the geodesic flow on the modular surface to our setting using the G_q-BCZ and symmetric G_q-Farey maps. This talk is self-contained and does not assume any prior knowledge of Hecke triangle groups or homogeneous dynamics.

Rodrigo Treviño (University of Maryland, College Park) *What we talk about when we talk about renormalization.* The use of operator algebras and their invariants in the study of minimal systems goes back many decades, producing along the way the remarkable result of Giordano, Putnam and Skau which classifies minimal \mathbb{Z} actions on Cantor sets up to orbit equivalence through the K-theory of certain operator algebras. In this talk I will talk about the trace cocycle, which is defined from invariants of the algebras lurking in the shadows of minimal systems. This yields new approach to the study of minimal, renormalizable \mathbb{R}^d actions and gives information about the deviation of ergodic averages through the Lyapunov spectrum of the trace
cocycle. I will mention the results obtained for translation flows on flat surfaces (joint with I. Putnam), as well as for the study of higher rank actions defined from aperiodic tilings.

**Kurt Vinhage** (Penn State University) *Classification of Cartan actions of abelian groups.* We will discuss recent progress on the Katok-Spatzier conjecture, which aims to classify Anosov $\mathbb{R}^k$ and $\mathbb{Z}^k$ actions under the assumption that there are no nontrivial smooth rank one factors. Classification is the strongest conclusion in the smooth rigidity program, which assumes nothing about the structure of the underlying manifold or dynamics other than the Anosov property. We develop new techniques to build homogeneous structures from dynamical ones. The remarkable features of the techniques are their low regularity requirements and their use of metric geometry over differential geometry to build group actions. We apply these techniques to obtain a classification result in the totally Cartan setting, where bundles associated to the hyperbolic structure are one-dimensional. Joint with Ralf Spatzier.

**Zhenqi Wang** (Michigan State University) *New progress of local rigidity of exceptional algebraic actions.* I will talk about the recent progress of the study of rigidity properties of exceptional algebraic actions. Exceptional algebraic actions possess geometric properties totally different from existing examples. We show $C^\infty$ local rigidity for a broad class of exceptional algebraic actions, including rank one partially hyperbolic actions, parabolic actions, higher rank but not genuinely higher rank partially hyperbolic actions. The method of proof is a combination of KAM type iteration scheme and representation theory and harmonic analysis. The principal difference with previous work that used KAM scheme is very general nature of the proof.

**Kitty Yang** (Northwestern University) *Mapping class group of minimal subshifts.* Let $(X,\sigma)$ be a subshift and $\text{Aut}(X)$ be the automorphism group, the group of self conjugacies of $(X,\sigma)$. The mapping class group, denoted $\mathcal{M}(\sigma)$, is the group of self flow equivalences. We show that $\mathcal{M}(\sigma)$ is constrained in the case of low-complexity minimal subshifts, similar to constraints on $\text{Aut}(X)$. In particular, when $(X,\sigma)$ is a minimal subshift associated to a substitution, $\mathcal{M}(\sigma)$ is an extension of $\mathbb{Z}$ by some finite subgroup of $\text{Aut}(X)$.

**Jing Zhou** (University of Maryland, College Park) *A rectangle billiard with moving slits.* In this talk, I will describe a two-dimensional exponential Fermi accelerator, which is realized as a rectangular billiard with moving slits. I will present the mechanism of “trapping regions”, which provides the desired exponential acceleration for almost all high energy orbits eventually. Explicit estimate on the waiting time is also available with an additional hyperbolicity assumption, after which most high energy orbits start exponential acceleration.