Instructions: To pass the exam you must correctly solve at least two of the following four problems. Your solutions will be evaluated for correctness, completeness and clarity. You must justify carefully any argument your use. Correct answers without supporting proof will be given no credit. You may use standard results without proof, provided that you state them clearly and completely (e.g. stating the name of a theorem is not sufficient). If you have any question about whether a particular result may be used without proof, please ask the faculty member proctoring the exam.

1. Is $S^4$ a covering space of $\mathbb{C}P^2$? Prove your answer.

2. Let $f : \mathbb{R}P^3 \to \mathbb{R}P^2$ be a continuous map. Compute the induced cohomology homomorphisms $f^* : H^i(\mathbb{R}P^2, \mathbb{Z}_2) \to H^i(\mathbb{R}P^3, \mathbb{Z}_2)$ for $i = 1, 2$.

3. Let $A$ be a diameter of the sphere $S^2$. Compute $H_i(S^2 \cup A)$ for all $i \geq 0$.

4. Prove that $S^4$ is not a topological group.