MATH 310 Course Objectives

Upon successful completion of MATH 310, the student should be able to:

- Apply the addition, subtraction, multiplication, and division principles to solve counting problems.
- Apply inclusion/exclusion to solve counting problems.
- Understand the concept of a permutation from a combinatorial perspective.
- Use permutations to solve problems involving arrangements of distinct objects.
- Understand the concept of a combination from a combinatorial perspective.
- Use combinations to solve problems involving selections of distinct objects.
- Understand the relationship between \( P(n,r) \) and \( C(n,r) \) where \( P(n,r) \) is the number of \( r \)-permutations of an \( n \)-element set and \( C(n,r) \) is the number of \( r \)-combinations of an \( n \)-element set.
- Use permutations to solve problems involving arrangements of elements of a multiset.
- Use combinations to solve problems involving selections of elements of a multiset.
- Prove Pascal's formula: \( C(n,r) = C(n-1,r-1) + C(n-1,r) \)
- Prove the Binomial Theorem.
- Use the Binomial Theorem to derive identities involving binomial coefficients.
- Prove binomial identities combinatorially.
- Model a combinatorial problem using recurrences.
- Find the generating function of a given sequence of numbers.
- Solve homogeneous recurrence relations using the roots of the characteristic equation of the recurrence.
- Solve homogeneous recurrence relations using generating functions.
- Solve nonhomogeneous recurrence relations using the roots of the characteristic equation of the recurrence.
- Solve nonhomogeneous recurrence relations using generating functions.
- Use combinatorial tools (such as recurrence relations and generating functions) to prove identities involving Stirling numbers of the first and second kinds.
- Use combinatorial tools (such as recurrence relations and generating functions) to prove identities involving partition functions.
MATH 311W Course Objectives

MATH 311W fulfills the departmental requirements for a writing course. The writing requirements in the course should complement the goal of development of mathematical communications skills.

Upon successful completion of MATH 311W, the student should be able to:

- Learn elementary propositional logic and set theory.
- Learn functions, one-to-one, onto, and 1-1 correspondence.
- Understand the concept of a relation and common examples of relations.
- Use deduction to prove mathematical theorems.
- Use induction to prove mathematical theorems.
- Use strong induction to prove mathematical theorems.
- Learn recursive definitions or recursion.
- Learn basic counting techniques including permutations and combinations.
- Learn the pigeonhole principle.
- Learn the binomial theorem.
- Learn the division algorithm.
- Understand greatest common divisors (GCDs).
- Learn the Euclidean algorithm and be able to use it to find a GCD.
- Learn modular arithmetic.
- Solve linear congruences.
- Solve simultaneous linear congruences using the Chinese remainder theorem.
- Learn Euler’s totient function.
- Learn Fermat’s little theorem and Euler’s theorem.
- Learn public-key cryptography.
- Learn the symmetric groups.
- Determine if a set is a group.
- Determine if a subset of a group is a subgroup.
- Learn group order.
- Understand the definition of coset.
- Learn Lagrange’s theorem.
- Learn group isomorphism.
MATH 312 Course Objectives

Upon successful completion of MATH 312, the student should be able to:

- State definitions rigorously, including those for limsup, liminf and limit of a sequence; convergent series; Cauchy sequence; subsequence; continuity at a point; and uniform continuity.
- Give rigorous proofs directly, by contradiction, induction or using counterexamples, in the context of real analysis.
- Give rigorous epsilon-delta proofs (e.g. that a quadratic polynomial is continuous at x=1).
- State theorems rigorously, including classical results such as the Bolzano-Weierstrass and Intermediate Value theorems.
- Articulate preliminary concepts in the real line (e.g. increasing/decreasing sequences, bounded sets, monotone convergence theorem).
- Quote standard facts, examples and counterexamples in sequences and series, such as giving examples of convergent and non-convergent sequences, knowing the harmonic series diverges while geometric series converge, and other convergence tests and examples.
- Formalize concepts such as sequences, convergence, limits and series, particularly power series, for functions (as opposed to numbers).
MATH 403 Course Objectives

Upon successful completion of MATH 403, the student should:

- Understand the basic notions of sets, relations, and functions. Know what natural numbers, integers, rational and real numbers are.
- Acquire understanding about finite, infinite, and countable sets. Be able to prove that there are uncountably many real numbers. Understand the axiom of completeness.
- Know the definition of a sequence and the definition of a convergent sequence. Be able to prove rules for limits of sums, products, and be able to use them.
- Know the Bolzano-Weierstrass Theorem, and the definition of a Cauchy sequence.
- Know and be able to use the Cauchy Criterion for convergence.
- Know the definition of convergence of a series in terms of partial sums, and be able to apply various tests for convergence and divergence.
- Understand the definitions of limit and continuity of a function. Be able to calculate limits and prove continuity of functions.
- Know the statements and proofs the extreme value theorem and the intermediate value theorem. Understand the difference between the notion of uniform continuity and continuity of a function. Know and be able to use the main properties of uniformly continuous functions.
- Acquire the knowledge about the pointwise and uniform convergence of sequences of functions. Be able to use the Weierstrass test to determine uniform convergence of series of functions.
- Know the definition of the derivative of a function. Be able to prove that a differentiable function is continuous. Be able to prove the rule of differentiation and be able to apply them.
- Know the definition of higher order derivatives. Understand the notion of an analytic function.
- Understand the definition of the Riemann integral and know its properties. Be able to give examples of integrable and non-integrable functions.
- Know the statements of the Fundamental Theorem of Calculus. Be able to use basic techniques of integration.
- Develop understanding about integration of sequences and series of functions.
- Know the notion of a metric space, inner product space, normed space, and topological space. Understand differences between them and be able to give examples of each of these spaces.
- Know the definition a continuous mapping between topological spaces and equivalent characterizations of continuity.
- Know the notion of a complete metric space. Be apply to apply the Contraction Mapping Theorem and the Baire’s Category Theorem.
- Know the notion of a compact topological space. Understand the characterization of compactness in finite dimensional Euclidean spaces. Develop appreciation of the role of compactness in studying continuity. Know and be able to use the
Arzela-Ascoli Theorem.
MATH 435 Course Objectives

In what follows, we hope that a student will demonstrate understanding of a number of definitions. We imagine this could be accomplished in some/all of the following ways.

- Have the ability to state the definition.
- Give an example or non-example of the definition.
- Be able to distinguish examples from non-examples.
- Prove that given examples satisfy, or do not satisfy, the given definition.
- Perform proofs involving abstract concepts which require an understanding of the definition.

We expect that performing proofs as noted in the last bullet above will be the most common method of demonstrating understanding.

Upon successful completion of MATH 435, the student should be able to:

- Demonstrate understanding of the definition of a group.
- Demonstrate understanding of the concept of cyclic groups.
- State and apply Lagrange's theorem.
- Demonstrate understanding of the concept of Abelian groups.
- Perform computations in common groups: for example groups of permutations, cyclic groups, dihedral groups and general linear groups.
- Demonstrate an understanding of subgroups including the special place of normal subgroups.
- Perform computations in quotient groups.
- Demonstrate understanding of the definition of a group homomorphism.
- State and/or use a version of the first isomorphism theorem for groups (and/or also for rings).
- Demonstrate understanding of conjugacy classes of elements within a group.
- Be able to prove simple statements for groups, including some of the above notions.
- Demonstrate understanding of the definition of a ring.
- Demonstrate understanding of the definition of an integral domain.
- Demonstrate understanding of the definition of a field.
- Demonstrate understanding of the concept of ideals, including prime and maximal ideals.
- Perform computations in quotient rings.
- Demonstrate understanding of the concept of principal ideal domains, especially with regards to \( k[x] \) and the set of integers.
- Use various tests to determine whether a polynomial with integer (or rational) coefficients is irreducible.
- Be able to construct extension fields, for example finite fields.
- Perform computations within extension fields.
- Demonstrate the ability to write simple abstract proofs for rings and fields which
utilize some of the above concepts.