**PHD QUALIFYING EXAMINATIONS**

**TOPOLOGY**

**December 10, 2017**

**Instructions:** To pass the exam you must correctly solve at least two of the following four problems. Your solutions will be evaluated for correctness, completeness and clarity.

You must justify carefully any argument your use. Correct answers without supporting proof will be given no credit. You may use standard results without proof, provided that you state them clearly and completely (e.g. stating the name of a theorem is not sufficient). If you have any question about whether a particular result may be used without proof, please ask the faculty member proctoring the exam.

**Note:** in the following problems, $\mathbb{T}^n$ denotes the standard $n$-dimensional torus.

1. Show that there are embeddings of two solid tori $A, B$ in $S^3$ such that $S^3$ is equal to the union of $A$ and $B$ with their boundaries, which are diffeomorphic to $\mathbb{T}^2$, identified.

2. Give a topological proof of the Fundamental Theorem of Algebra: every non-constant complex polynomial has a root.

3. Consider a $2 \times 2$ matrix $A$ with integer entries. This matrix determines a map $f$ from the torus $\mathbb{T}^2$ identified with $\mathbb{R}^2/\mathbb{Z}^2$ to itself, given by $f(v) = Av \mod \mathbb{Z}^2$ if $v \in \mathbb{R}^2 \mod \mathbb{Z}^2$. Find the degree of the map $f$.

4. Find the fundamental group of the punctured torus, that is, of $\mathbb{T}^2$ with a point deleted.