PROGRAM AND ABSTRACTS

Synergistic Room: the big room on the ground floor of the McAllister Building (to the left of the math statue).

MASS Room: the neighboring classroom to the right of the math statue.

All talks, unless otherwise indicated, will take place in the synergistic room.

THURSDAY 10/23

1:00-2:00. Registration

2:00-2:45. Hasselblatt: Rigidity of Anosov flows

3:00-3:45. Kalinin: Livsic Theorem for matrix cocycles

3:45-4:15. Coffee break

4:15-5:00. Hochman: The ratio ergodic theorem for multiparameter actions

5:15-6:00. Damjanovic: Perturbation results for some parabolic algebraic actions

FRIDAY 10/24

8:30-9:20. Registration

9:20-9:30. Greeting by the chairman

9:30-10:15. Oh: Apollonian circle packings and long horospheres in infinite volume hyperbolic 3 manifolds

10:30-11:15. Gorodnik: Rigidity of actions on algebraic spaces

11:15-11:45. Coffee break

11:45-12:30. Einsiedler: Unipotent joinings in positive characteristic

12:30-2:00. Lunch break

2:00-3:00. Young (Colloquium): Shear-induced chaos

3:00-3:30. Coffee break

3:30-4:15 Zhang: Decay of correlations for hyperbolic systems with singularities

3:30-4:15 Gogolev: Smooth conjugacy of Anosov systems on higher dimensional tori

4:30-5:15 Todd: Multifractal analysis for multimodal maps

4:30-5:15 Fish: Measure rigidity of an action of a multiplicative semigroup of transformations of polynomial growth on the torus

Date: October 23, 2008.
Saturday 10/25

8:30-9:00. Registration

9:00-9:45. Forni: *Quantitative equidistribution of nilflows and Weyl sums*

10:00-10:45. Damanik: *Lyapunov Exponents of Schrödinger Cocycles and the Kotani Last Conjecture*

10:45-11:15. Coffee break

11:15-12:00. Dolgopyat: *Semicircular outer billiards*

12:00-1:30. Lunch break

1:30-2:15. Shah: *Limits of translates of curves on homogeneous spaces and Dirichlet’s theorem on Diophantine approximation*


3:15-3:45. Coffee break

**MASS Room**

3:45-4:10. Cyr: *Topology of Potentials on Countable Markov Shifts*

3:45-4:10. Nanes: *On the Ergodicity of Partially Hyperbolic Group Actions*

4:20-4:45. Sun: *Zero entropy invariant measures for skew product diffeomorphisms*

4:20-4:45. Webb: *Dynamics of Functions with an Eventual Negative Schwarzian Derivative*

4:55-5:20. McGoff: *Symbolic extensions, Cantor-Bendixon rank, and maps of the interval*

4:55-5:20. Thompson: *The Liouville entropy of a 3-manifold is not monotonic along the Ricci flow*

8:00. Party at Y. Pesin’s house

Sunday 10/26

9:00-9:45. Burns: *Recent advances in partial hyperbolicity*

10:00-10:45. Sotomayor: *Surfaces in \( \mathbb{R}^3 \) with dense principal curvature lines*

10:45-11:15. Coffee break

11:15-12:00. Orshanskiy: *A PL-manifold of nonnegative curvature homeomorphic to \( S^2 \times S^2 \) is a direct metric product*

12:15-1:00. Nabutovsky: *Local minima of the length functional on loop spaces*
**Bergelson:** Ergodic theory and combinatorics in finite characteristic.

Let $F = \mathbb{Z}/p\mathbb{Z}$ be the cyclic group on $p$ elements, where $p$ is a prime and let $F^\infty$ denote the direct sum of countably many copies of $F$. We shall discuss some old and new results pertaining to measure preserving actions of $F^\infty$ and applications thereof to algebra and combinatorics. We shall also formulate and discuss some natural open problems and possible new applications.

**Burns:** Recent advances in partial hyperbolicity

I will attempt to survey recent developments in the study of partially hyperbolic diffeomorphisms.

**Cyr:** Topology of Potentials on Countable Markov Shifts

A Markov shift is the left shift map acting on the set of infinite trajectories through a directed graph. It is known in thermodynamic formalism for finite Markov shifts (graphs with finitely many vertices), that there is always an equilibrium measure. In the case of a countable Markov shift the situation is not as simple; there may (and frequently do) exist potentials that have no equilibrium measure. In this talk we consider the space of good potentials on a countable Markov shift in several natural topologies. We show that the set of potentials with a certain generalization of an equilibrium measure contains a $\| \cdot \|_\infty$–open and $\| \cdot \|_{\text{Lip}}$–dense set. This is joint work with Omri Sarig.

**Damanik:** Lyapunov Exponents of Schrödinger Cocycles and the Kotani-Last Conjecture

The Kotani-Last conjecture is a central open problem in the theory of ergodic Schrödinger operators on the line. In spectral terms it states that the presence of absolutely continuous spectrum implies the almost periodicity of the potentials. There is an equivalent dynamical formulation of this conjecture in terms of the Lyapunov exponents of the associated Schrödinger cocycles. This talk will discuss the history of this conjecture, known partial results, and some specific obstructions to a complete resolution.

**Damjanovic:** Perturbation results for some parabolic algebraic actions

I will discuss local structure for unipotent-generated rank-two abelian actions on homogeneous spaces $X/L$, where $X$ is $\text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{K})$, with $\mathbb{K}$ real or complex, and $L$ a cocompact irreducible lattice. Unlike their rank-one counterparts (horocycle flows) these rank-two actions have vanishing first cohomology, which implies a weak local classification.

**Demers:** Billiards with Holes

Introducing a small hole into the phase space of an ergodic dynamical system causes almost every trajectory to eventually escape. Despite this, such systems can have rich dynamics. For dispersing billiards with holes, we construct physically relevant invariant and conditionally invariant measures with properties analogous to those of SRB measures for closed systems. We prove a variational principle involving the escape rate and show that the conditionally invariant measures converge to the smooth invariant measure in the small hole limit. This is joint work with Lai-Sang Young and Paul Wright.

**Dolgopyat:** Semicircular outer billiards.

We show that the semicircular outer billiards has an unbounded orbit. We then discuss related results and open questions for the dynamics of piecewise smooth transformations.

**Einsiedler:** Unipotent joinings in positive characteristic

We will discuss joinings of horospherical subgroups (in positive characteristic) and describe how to derive the following theorem from it. Let $X$ be the quotient of, say for simplicity, $G = \text{SL}(n, k)$ by a lattice $\Gamma$. Any diagonal $G$-orbit in $X^2$ is either dense or closed. Clearly the mentioned results are special cases of Ratner’s celebrated theorems, but for $k$ having positive characteristic, this is joint work with Amir Mohammadi.
**Fish:** Measure rigidity of an action of a multiplicative semigroup of transformations of polynomial growth on the torus

We prove by use of entropy technique that any invariant ergodic measure under the action of such semigroup is either Lebesgue or has a finite support.

**Forni:** Quantitative equidistribution of nilflows and Weyl sums

It is known that the equidistribution of the fractional parts of polynomials sequences with irrational leading coefficient can be derived from the unique ergodicity of nilpotent maps on the torus or, equivalently, of homogeneous flows on nilmanifolds. We will present some results on the speed of convergence of ergodic averages of nilflows under Diophantine conditions and discuss the relation with known results and conjectures on Weyl sums (exponential sums for polynomial sequences). Our approach is based on methods from the theory of dynamical systems (cohomological equations, invariant distributions, renormalization). The content of this talk is joint work with L. Flaminio (Lille).

**Gogolev:** Smooth conjugacy of Anosov systems on higher dimensional tori

Structural stability asserts that if two Anosov diffeomorphisms are close enough then they are topologically conjugate: \( h f = g h \). There are simple obstructions to the smoothness of \( h \). If \( x \) is a periodic point of \( f \) with period \( p \) then \( h(x) \) is periodic for \( g \). If \( h \) is differentiable, then the differentials \( D(f^p)(x) \) and \( D(g^p)(h(x)) \) would be conjugate by the differential of \( h \). Thus spectral data at periodic points are moduli of \( C^1 \) conjugacy.

Question: Suppose that the periodic data of \( f \) and \( g \) coincide. Is then \( h \) differentiable? If it is, how smooth is it?

For Anosov diffeomorphisms of 2-torus, the complete answer to this question was given by de la Llave, Marco and Moriyon (87-90).

We study the smooth conjugacy problem in a small \( C^1 \)-neighborhood of hyperbolic automorphisms of \( d \)-torus \( \mathbb{T}^d, d > 2 \).

**Gorodnik:** Rigidity of actions on algebraic spaces

We survey some of the known rigidity results which show that objects defined in the measurable category are algebraic and discuss a joint work with Bader, Furman, and Weiss that classifies measurable isomorphisms and factors for ergodic actions on homogeneous varieties.

**Hasselblatt:** Rigidity of Anosov flows

For a uniformly quasiconformal transversely symplectic Anosov flow we define the *longitudinal KAM-cocycle* and use it to prove a rigidity result: \( E^u \oplus E^s \) is Zygmund-regular, and higher regularity implies vanishing of the longitudinal KAM-cocycle, which in turn implies that the flow is smoothly conjugate to an algebraic one.

**Hochman:** The ratio ergodic theorem for multiparameter actions

The ergodic theorem has been generalized to a very wide class of groups. In contrast, Hopf’s ratio ergodic theorem, which deals with integer actions on infinite probability spaces, was until recently not known even for \( \mathbb{Z}^2 \)-actions, and was believed not to hold, due to a counter-example of Brunel and Krengel. Although this example is correct it is for the "wrong" kind of averages, and a few years ago J. Feldman proved a partial result for \( \mathbb{Z}^d \) actions under the assumption that the generators of the actions are conservative as individual transformations. I’ll discuss an unconditional proof of the Hopf theorem for \( \mathbb{Z}^d \) actions which relies on some delicate geometric properties of \( \mathbb{R}^d \), and why it will be more difficult to extend this result to other groups.
Kalinin: Livsic Theorem for matrix cocycles.

We prove the Livsic periodic point theorem for arbitrary $GL(m, \mathbb{R})$ cocycles. We consider a hyperbolic dynamical system $f : X \to X$ and a Hölder continuous function $A : X \to GL(m, \mathbb{R})$. We show that if $A$ has trivial periodic data, i.e. $A(f^{n-1}p) \cdots A(fp)A(p) = Id$ for each periodic point $p = f^n p$, then there exists a Hölder continuous function $C : X \to GL(m, \mathbb{R})$ satisfying $A(x) = C(fx)C(x)^{-1}$ for all $x \in X$. The main new ingredients in the proof are some results of independent interest on relations between the periodic data, Lyapunov exponents, and uniform estimates on growth of products along orbits for an arbitrary Hölder function $A$.

McGoff: Symbolic extensions, Cantor-Bendixon rank, and maps of the interval

Using the theory of Symbolic Extensions and Entropy Structures developed by Boyle and Downarowicz, one may define the order of accumulation of symbolic extension entropy for a dynamical system, which will be an invariant of topological conjugacy. We discuss the relationship between this order of accumulation and certain topological invariants of the simplex of invariant measures, while also providing a family of examples using maps of the interval. This talk discusses joint work with David Burguet.

Nabutovsky: Local minima of the length functional on loop spaces

Let $M$ be a closed simply connected Riemannian manifold, and $p$ an arbitrary point of $M$. We demonstrate that if the length functional on the space of loops on $M$ based at $p$ has a very deep non-trivial local minimum, then it has many deep local minima. To prove this result we introduce a notion of “effective universal covering”, which has other potential applications in Riemannian geometry.

Nanes: On the Ergodicity of Partially Hyperbolic Group Actions

In 1995, Pugh and Shub conjectured that ergodicity is $C^r$ open and dense in the space of $C^r$ partially hyperbolic diffeomorphisms. Much work has gone into proving this conjecture; this discussion will talk about generalizations of the latest results to the setting of partially hyperbolic flows, and more generally, group actions.

Oh: Apollonian circle packings and long horospheres in infinite volume hyperbolic 3 manifolds

Apollonian circle packings are arrangements of tangent circles that arise by repeatedly filling the interstices between four mutually tangent circles with further tangent circles.

In a recent joint work with Alex Kontorovich, we obtain the asymptotic number of circles of curvature at most $T$ (as $T$ tends to infinity) in any given Apollonian packing. We also obtain the optimal upper bounds for the number of primes/twin primes in any ”integral” Apollonian packing.

In my talk, I will try to explain these results as well as to explain how the equidistribution of long closed horospheres in infinite volume hyperbolic 3 manifolds are related.

Orshanskiy: A PL-manifold of nonnegative curvature homeomorphic to $S^2 \times S^2$ is a direct metric product

Let $M^4$ be a PL-manifold of nonnegative curvature that is homeomorphic to a product of two spheres, $S^2 \times S^2$. We prove that $M$ is a direct metric product of two spheres endowed with some polyhedral metrics. In other words, $M$ is a direct metric product of the surfaces of two convex polyhedra in $\mathbb{R}^3$. The background for the question is the following. The classical H.Hopf’s hypothesis states: for any Riemannian metric on $S^2 \times S^2$ of nonnegative sectional curvature the curvature cannot be strictly positive at all points. There is no quick answer to this question: it is known that a Riemannian metric on $S^2 \times S^2$ of nonnegative sectional curvature need not be a product metric. However, M.Gromov has pointed out that the condition of nonnegative curvature in the PL-case appears to be stronger than nonnegative sectional curvature of Riemannian manifolds and analogous to some condition on the curvature operator. The result presented in this talk settles the PL-version of the Hopf’s hypothesis.
**Shah:** Limits of translates of curves on homogeneous spaces and Dirichlet’s theorem on Diophantine approximation

We show that the Dirichlet’s theorem on simultaneous diophantine approximation cannot be improved in the sense of Davenport and Schmidt for almost all points on an analytic curve in $\mathbb{R}^k$. The result is a consequence of an equidistribution theorem about limits of certain translates of curves on homogeneous spaces of $\text{SL}(n, \mathbb{R})$. The proof involves Ratner’s theorem on classification of ergodic invariant measures for unipotent flows and intertwined dynamics of linear actions of $\text{SL}(n, \mathbb{R})$’s on vector spaces.

**Sotomayor:** Surfaces in $\mathbb{R}^3$ with dense principal curvature lines

In [1,2] it was proved that every compact, oriented, smooth surface, $M(g)$, where $g$ denotes the genus, can be arbitrarily $C^2$ approximated by one with no recurrent lines of principal curvature. Explicit examples were given with dense lines of curvature in cases $g = 0$, spheroidal, and $g = 1$, toroidal surfaces.

In this talk, examples for the cases $g \geq 2$, “pretzels”, will be provided.


**Sun:** Zero entropy invariant measures for skew product diffeomorphisms

For skew product diffeomorphisms with some nonuniform hyperbolic structure along fibers, we are able to find an invariant measure of zero entropy. We can take some proper Pesin set and a measurable section with controlled return time to the Pesin set. By considering the return map and its projection to the base, we can find some good pseudo orbits, from which we can construct an invariant set with shadowing technique. This set then gives the measure whose conditional measure on each fiber is a counting measure.

**Thompson:** The Liouville entropy of a 3-manifold is not monotonic along the Ricci flow

In 2004, Manning used an important formula of Katok, Knieper and Weiss to prove that as the metric on a negatively curved surface evolves under the (normalised) Ricci flow, the topological entropy of the geodesic flow decreases.

In contrast, we observe that an example of Flaminio can be used to show that the Liouville entropy can both increase or decrease along a Ricci flow in a neighbourhood of a particular 3-manifold of constant negative curvature.

We will introduce this topic, explain our observation, and recall some of the interesting and challenging open questions in this area.

**Todd:** Multifractal analysis for multimodal maps

Let $f$ be a multimodal map of the interval $I$. Given an equilibrium state $\mu_\phi$ for a Hölder potential $\phi : I \to \mathbb{R}$, the local dimension $d_{\mu_\phi}(x)$ measures how concentrated $\mu_\phi$ is at this point. The dimension spectrum encodes the Hausdorff dimension of level sets of $d_{\mu_\phi,bi}$. This spectrum can be understood via induced maps $(X, F)$, where $F = f^\tau$ for some inducing time $\tau$. A major challenge for maps with critical points is to find inducing schemes which “see” a sufficiently large subset of the space. In this talk I will explain that this problem can be overcome, and hence that, as in the uniformly expanding case, the dimension spectrum is encoded by a function related to the pressure of some potentials involving $\phi$. Many of these results apply not only to Collet-Eckmann maps, but also to maps with much weaker growth conditions.
**Young:** Shear-induced chaos

By shear-induced chaos in a driven dynamical system, I refer to the phenomenon in which a system with simple dynamics acquires sustained chaotic behavior when the effect of periodic forcing is amplified by shearing in the system. Two mathematical characterizations of chaos, namely the instability of large sets of orbits and the resemblance of dynamical data to those generated by stochastic processes, will be discussed, and known ways of proving these properties will be reviewed briefly. The focus of much of this talk will be on shear as a geometric mechanism for producing chaos. I will demonstrate that this phenomenon manifests itself in many different guises, from kicked limit cycles to Hopf bifurcations to coupled oscillators, in systems defined by ODEs, SDEs and PDEs. A combination of analytical and numerical results will be presented.

**Webb:** Dynamics of Functions with an Eventual Negative Schwarzian Derivative

In the study of one dimensional dynamical systems it is often assumed that the functions involved have a negative Schwarzian derivative. However, as not all one dimensional systems of interest have this property, much work has gone into determining to what extent the dynamical properties exhibited by such functions extend to other classes of functions without a negative Schwarzian derivative. With this in mind we consider a generalization of this condition. Specifically, we consider the interval functions of a real variable having some iterate with a negative Schwarzian derivative and show that many known results generalize to this larger class, that is to functions with an eventual negative Schwarzian derivative. The property of having an eventual negative Schwarzian derivative is nonasymptotic therefore verification of whether a function has such an iterate can often be done by direct computation. The introduction of this class was motivated by some maps arising in neuroscience.

**Zhang:** Decay of correlations on hyperbolic systems with general singularities

We study hyperbolic systems with singularities and prove the coupling lemma and exponential decay of correlations under weaker assumptions than previously adopted in similar studies. Our new approach allows us to treat certain billiard models that could not be handled by the traditional techniques. These models include modified Bunimovich stadia, which are bounded by minor arcs, and flower-type regions that are bounded by major arcs. This is a work jointed with N. Chernov.