Marian Gidea (Northeastern Illinois University). Diffusion with optimal time in the large gap problem.

Abstract. We present a topological mechanism for diffusion in the large gap problem for Hamiltonian systems. We consider systems consisting of $n$ penduli and a rotator with a weak, periodic coupling, described by Hamiltonians of the form

$$\sum_{i=1}^{n} \pm \left( \frac{1}{2} p_i^2 + V_i(q_i) \right) + \epsilon h(p_1, \ldots, p_n, q_1, \ldots, q_n, I, \phi, t; \epsilon),$$

where $(p_1, \ldots, p_n, q_1, \ldots, q_n, I, \phi, t) \in \mathbb{R}^n \times \mathbb{T}^n \times \mathbb{R} \times \mathbb{T} \times \mathbb{T}$ with the standard symplectic structure. We assume that $V_i, h_0, h$ are $C^{k+1}$-differentiable ($k \geq 2$). We also assume that each $V_i$ is periodic in $q_i$ of period 1 and has a unique non-degenerate global maximum, that $h_0$ satisfies a uniform twist condition, and that the perturbation $h$ is periodic in $t$ of period 1.

We show that if the perturbation $h$ satisfies some explicit non-degeneracy conditions of Melnikov type, which are $C^{k+1}$-open and $C^\infty$-dense, then there exist trajectories $x(t)$ along which $|I(x(T)) - I(x(0))|$ is of order $O(1)$ with respect to $\epsilon$, for some time $T$ of order $O((1/\epsilon) \ln(1/\epsilon))$. There are known upper bounds for $|I(x(T)) - I(x(0))|$ which show that this time $T$ is optimal up to a constant.

The proof is based on the theory of normally hyperbolic manifolds and on the method of correctly aligned windows.