

## Section 5.4 Exponential and Logarithmic Equations

### Objective 1: Solving Exponential Equations

$$\text{If } u = v, \text{ then } \log_b u = \log_b v.$$

**Logarithm Property of Equality**

$$\log_b u^r = r \log_b u$$

**Power Rule for Logarithms**

#### Solving Exponential Equations

- If the equation can be written in the form  $b^u = b^v$ , then solve the equation  $u = v$ .
- If the equation cannot be written in the form  $b^u = b^v$ :
  1. Use the Logarithm Property of Equality to “take the log of both sides” (typically using base 10 or base  $e$ ).
  2. Use the Product Rule of Logarithms to “bring down” any exponents.
  3. Solve for the given variable.

**IMPORTANT NOTE:** Your MML homework will require a calculator. On quizzes and exams, you will leave your answer in “exact” form in simplified terms of logarithms.

5.5.1, 4, 7, 8, and 12

Solve the exponential equation.

1.  $3^x = 5$

4.  $3^{x+7} = -20$

7.  $8^{4x-7} = 11^{5+x}$

8.  $3(9^{x-1}) = 81^{2x+1}$

12.  $150e^{x-4} = 5$

## Objective 2: Solving Logarithmic Equations

### Properties of Logarithms

If  $b > 0$ ,  $b \neq 1$ ,  $u$  and  $v$  represent positive numbers, and  $r$  is any real number, then

$$\log_b uv = \log_b u + \log_b v \quad \text{Product Rule for Logarithms}$$

$$\log_b \frac{u}{v} = \log_b u - \log_b v \quad \text{Quotient Rule for Logarithms}$$

$$\log_b u^r = r \log_b u \quad \text{Power Rule for Logarithms}$$



**When solving logarithmic equations, it is important to always verify the solutions. Logarithmic equations often lead to extraneous solutions.**

When a logarithmic equation cannot be written in the form  $\log_b u = \log_b v$ , we follow the steps outlined below.

### Solving Logarithmic Equations

1. Determine the domain of the variable.
2. Use properties of logarithms to combine all logarithms and write as a single logarithm if needed.
3. Eliminate the logarithm by rewriting the equation in exponential form.
4. Solve for the given variable.
5. Check for any extraneous solutions. Verify that each solution is in the domain of the variable.

5.5.21, 26, and 29

Solve the logarithmic equation.

21.  $\log(1 - 5x) = 2$

26.  $\log_7(x + 9) = \log_7(x + 15) = 1$

29.  $\ln 3 + \ln \frac{x}{e} + \frac{2x}{3} = 0$