Section 5.3   Properties of Logarithms

Objective 1: Using the Product Rule, Quotient Rule and Power Rule for Logarithms

Properties of Logarithms

If $b > 0$, $b \neq 1$, u and v represent positive numbers, and r is any real number, then

$\log_b uv = \log_b u + \log_b v$ **Product Rule for Logarithms**

$\log_b \dfrac{u}{v} = \log_b u - \log_b v$ **Quotient Rule for Logarithms**

$\log_b u^r = r \log_b u$ **Power Rule for Logarithms**

CAUTION

Warning!

$\log_b (u+v)$ is NOT equivalent to $\log_b u + \log_b v$.

$\log_b (u-v)$ is NOT equivalent to $\log_b u - \log_b v$.

$\dfrac{\log_b u}{\log_b v}$ is NOT equivalent to $\log_b u - \log_b v$.

$(\log_b u)^r$ is NOT equivalent to $r \log_b u$.

5.4.4, 5, 7, and 9
Use the properties of logarithms to expand the expression. Wherever possible, evaluate the expression.

4. $\log_3 (27w)$ 5. $\ln 5e^2$ 7. $\log 100P$ 9. $\log_{\sqrt{2}} 8x$

Objective 2: Expanding and Condensing Logarithmic Expressions

5.4.13
Use the properties of logarithms to expand the expression. Wherever possible, evaluate the expression.

$\log \dfrac{\sqrt{x}}{10y^3}$

5.4.27
Use the properties of logarithms to rewrite the expression as a single logarithm. Wherever possible, evaluate the expression.

$\log_5(x - 2) + \log_5(x + 2)$

Objective 3: Solving Logarithmic Equations Using the Logarithm Property of Equality

The Logarithm Property of Equality

If a logarithmic equation can be written in the form $\log_b u = \log_b v$, then $u = v$.

Furthermore, if $u = v$, then $\log_b u = \log_b v$.

5.4.34
Use the properties of logarithms and the logarithm property of equality to solve the equation.

$\ln 5 + \ln x = \ln 7 + \ln(3x - 2)$

Objective 4: Using the Change of Base Formula

Change of Base Formula

For any positive base $b \neq 1$ and for any positive real number u, then

$$\log_b u = \frac{\log_a u}{\log_a b}$$

where a is any positive number such that $a \neq 1$.

5.4.37 First, *guess* the value based on your knowledge of logarithms, then use the change of base formula and a calculator to approximate the following expressions. Do not round until the final answer. Then round to four decimal places as needed.

$\log_4 51$

5.4.46
Solve the equation and simplify the answer.

$\log_{1/3} x = \log_3 20$