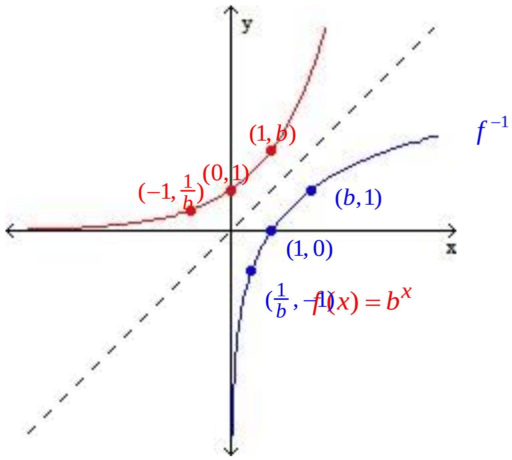
Section 5.2 Logarithmic Functions

Objective 1: Understanding the Definition of a Logarithmic Function

Every exponential function of the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$ is one-to-one and thus has an inverse function.

The graph of $f(x) = b^x$, $b > 1$ and its inverse.



To find the equation of f^{-1}:

Step 1. **Change $f(x)$ to y:** $y = b^x$

Step 2. **Interchange x and y:** $x = b^y$

Step 3. **Solve for y:** ??

Before we can solve for y we must introduce the following definition:

Definition of the Logarithmic Function

For $x > 0$, $b > 0$ and $b \neq 1$, the **logarithmic function with base *b*** is defined by

$$y = \log_b x \text{ if and only if } x = b^y.$$

Step 3. **Solve for y:** $x = b^y$ can be written as $y = \log_b x$

Step 4. **Change y to $f^{-1}(x)$:** $f^{-1}(x) = \log_b x$

5.3.1
Write the exponential equation as an equation involving a logarithm.
$3^2 = 9$

5.3.8
Write the logarithmic equation as an exponential equation.
$\log_7 343 = 3$

Objective 2: Evaluating Logarithmic Expressions

The expression $\log_b x$ is the exponent to which b must be raised to in order to get x.

5.3.12
Evaluate the logarithm without the use of a calculator.
$\log_2 8$

Objective 3: Understanding the Properties of Logarithms

General Properties of Logarithms
For $b > 0$ and $b \neq 1$,
(1) $\log_b b = 1$ and
(2) $\log_b 1 = 0$.

Cancellation Properties of Exponentials and Logarithms
For $b > 0$ and $b \neq 1$,
(1) $b^{\log_b x} = x$ and
(2) $\log_b b^x = x$.

5.3.21 and 24
Use the properties of logarithms to evaluate the expression without the use of a calculator.

21. $\log_9 1$

24. $5^{\log_5 M}, M > 0$

Objective 4: Using the Common and Natural Logarithms

Definition of the Common Logarithmic Function

For $x > 0$, the **common logarithmic function** is defined by

$$y = \log x \text{ if and only if } x = 10^y.$$

(Think: $\log_{10} x = \log x$)

Definition of the Natural Logarithmic Function

For $x > 0$, the **natural logarithmic function** is defined by

$$y = \ln x \text{ if and only if } x = e^y.$$

(Think: $\log_e x = \ln x$)

5.3.27, 28
Write the exponential equation as an equation involving a common logarithm or a natural logarithm.

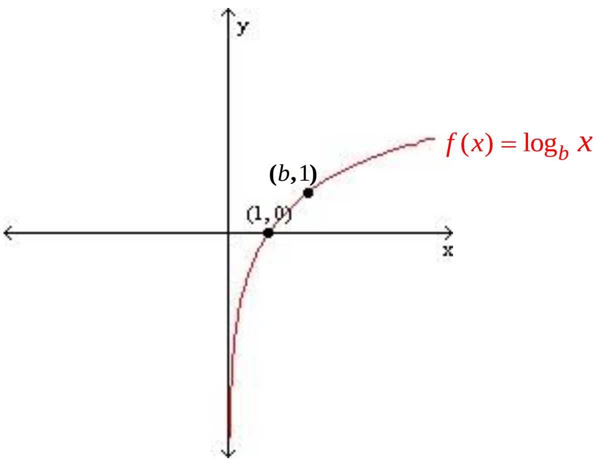
27. $10^3 = 1,000$

28. $e^{-1} = \frac{1}{e}$

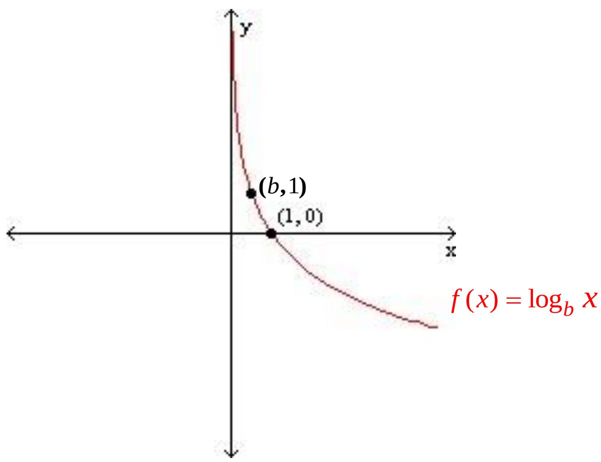
Objective 5: Understanding the Characteristics of Logarithmic Functions

Characteristics of Logarithmic Functions
For $b > 0$, $b \neq 1$ the logarithmic function with base b is defined by $y = \log_b x$.

The domain of $f(x) = \log_b x$ is $(0, \infty)$ and the range is $(-\infty, \infty)$. The graph of $f(x) = \log_b x$ has one of the following two shapes.



$f(x) = \log_b x$, $b > 1$



$f(x) = \log_b x$, $0 < b < 1$

5.3.32, 33
Write the logarithmic equation as an exponential equation.

32. $\ln 1 = 0$

33. $\log(1,000,000) = 6$

5.3.37, 40
Evaluate the expression without the use of a calculator.

37. $\log\left(\frac{1}{1,000}\right)$

40. $10^{\log e}$

Objective 6: Sketching the Graphs of Logarithmic Functions Using Transformations

5.3.49
Sketch the logarithmic function. Label at least 2 points on the graph and determine the domain and the equation of any vertical asymptotes.

$y = \log_{1/2}(x+1)+2$

Objective 7: Finding the Domain of Logarithmic Functions

If $f(x) = \log_b\left[g(x)\right]$, then the domain of f can be found by solving the inequality $g(x) > 0$.

5.3.54, 55
Find the domain of the logarithmic function.

54. $f(x) = \ln(1 - 3x)$

55. $f(x) = \log_2(x^2 - 9)$