Section 4.3 The Graphs of Polynomial Functions

Objective 1: Understanding the Definition of a Polynomial Function

Definition Polynomial Function

The function $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$ is a polynomial function of degree n where n is a nonnegative integer. The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the **coefficients** of the polynomial function. The number a_n is called **the leading coefficient** and a_0 is called the **constant coefficient**.

4.3.1, 2, and 4
Determine if the given function is a polynomial function. If it is, then identify the degree, the leading coefficient, and the constant coefficient.

a) *f(x)* = ____________________________

Degree ________

Leading Coefficient ________

Constant Coefficient ________

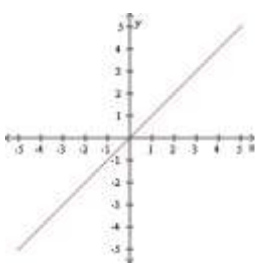
b) *f(x)* = ____________________________

Degree ________

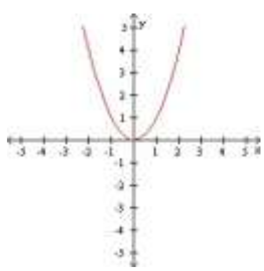
Leading Coefficient ________

Constant Coefficient ________

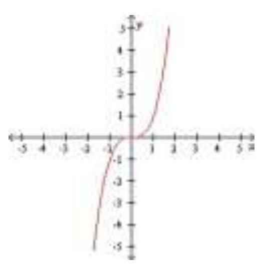
Objective 2: Sketching the Graphs of Power Functions



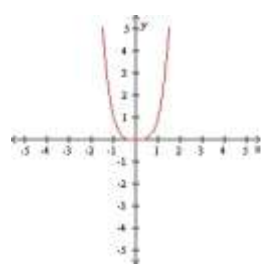
(a) $f(x) = x$



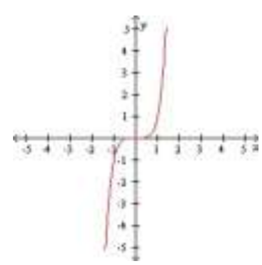
(b) $f(x) = x^2$



(c) $f(x) = x^3$



(d) $f(x) = x^4$



(e) $f(x) = x^5$

4.3.10 and 15
Use the associated power function and transformations to sketch the following functions.

a) *f(x)* = ______________________________

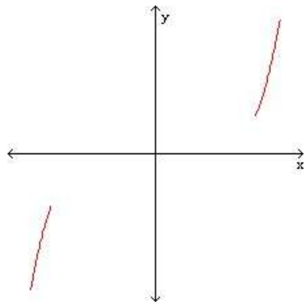
b) *f(x)* = ______________________________

Objective 3: Determining the End Behavior of Polynomial Functions

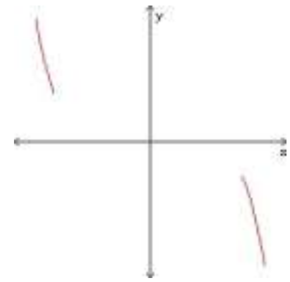
Process for Determining the End Behavior of a Polynomial Function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0 .$$

If the degree n is odd, the graph has opposite left-hand and right-hand end behavior, that is, the graph "starts" and "finishes" in **opposite** directions.

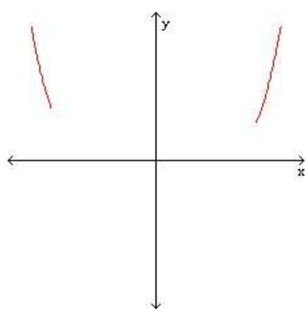


$a_n > 0$, odd degree

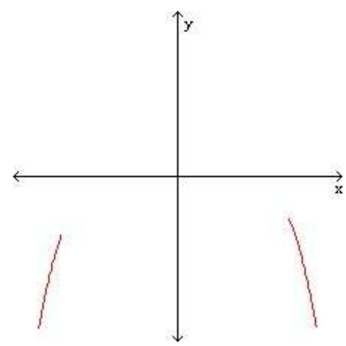


$a_n < 0$, odd degree

If the degree n is even, the graph has the same left-hand and right-hand end behavior, that is, the graph "starts" and "finishes" in the **same** direction.



$a_n > 0$, even degree



$a_n < 0$, even degree

4.3.16 and 21
Use the end behavior of the graph of the given polynomial function to answer the following:

16
a)The degree is ______________ (even or odd).

b) The leading coefficient is ______________ (positive or negative).

21
a)The degree is ______________ (even or odd).

b) The leading coefficient is ______________ (positive or negative).

Objective 4: Determining the Intercepts of a Polynomial Function

The number $x = c$ is called a **zero** of a function f if $f(c) = 0$. If c is a real number, then c is an x-intercept. Therefore, to find the x-intercepts of a polynomial function $y = f(x)$, we must find the real solutions of the equation $f(x) = 0$.

4.3.23
Find the intercepts of the polynomial function *f(x)* = ____________________________.

The *y*-intercept is *y* = ________.

The *x*-intercept(s) is/are *x* = ____________________.

4.3.25
Find the intercepts of the polynomial function *f(x)* = ____________________________.

The *y*-intercept is *y* = ________.

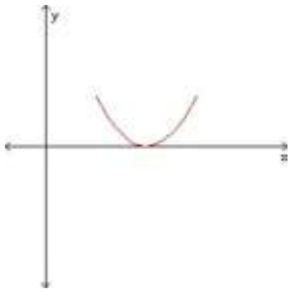
The *x*-intercept(s) is/are *x* = ____________________.

Objective 5: Determining the Real Zeros of Polynomial Functions and Their Multiplicities

The Shape of the Graph of a Polynomial Function Near a Zero of Multiplicity *k*.

Suppose c is a real zero of a polynomial function f of multiplicity k, that is, $(x-c)^k$ is a factor of f. Then the shape of the graph of f near $x = c$ is as follows:

If $k > 1$ is even, then the graph **touches** the x-axis at $x = c$.



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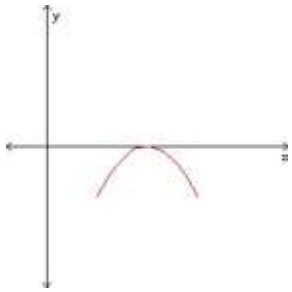
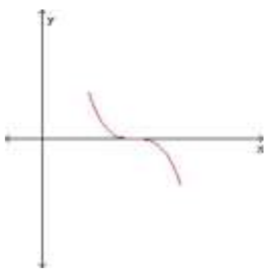


If $k > 1$ is odd, then the graph **crosses** the x-axis at $x = c$.



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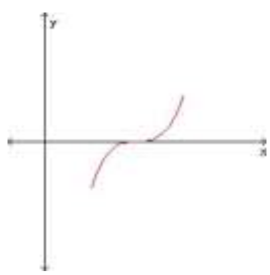

4.3.30
Determine the real zeros and their multiplicities of $f(x)$ = ____________________________.

a) The real zeros of the polynomial are x = ____________________. (Use a comma to separate answers as needed. Type an exact answer, using radicals as needed.)

b) The multiplicity of the zero located farthest left on the x-axis is _____.

The multiplicity of the zero located farthest right on the x-axis is _____.

The graph _________the x-axis at the leftmost zero. (touches or crosses)

The graph __________ the x-axis at the rightmost zero. (touches or crosses)

Objective 6: Sketching the Graph of a Polynomial Function

Four-Step Process for Sketching the Graph of a Polynomial Function

1. Determine the end behavior.
2. Plot the *y*-intercept $f(0) = a_0$.
3. Completely factor f to find all real zeros and their multiplicities*.
4. Choose a test value between each real zero and sketch the graph.

* This is the most difficult step and will be discussed in further detail in the subsequent sections of this chapter.

4.3.36

Sketch the polynomial function *f(x)* = ______________________________ using the four-step process.

The left-hand behavior starts _____ and the right-hand behavior ends _____.

The *y*-intercept is _____.

The real zeros of the polynomial are *x* = ____________________.

The multiplicity of the zero located farthest left on the *x*-axis is _____.

The multiplicity of the zero located between the leftmost and rightmost zeros is _____.

The multiplicity of the zero located farthest right on the *x*-axis is _____.

What is the value of the test point at *x* = _____? *y* = _____

Sketch the graph.

Objective 7: Determining a Possible Equation of a Polynomial Function Given its Graph

4.3.45
Analyze the graph to address the following about the polynomial function it represents.

a) Is the degree even or odd? ___________

b) Is the leading coefficient positive or negative? ___________

c) The value of the constant coefficient is _____.

d) The leftmost real zero is x = _____, which has an _____ multiplicity.

The second real zero from the left is x = _____, which has an _____ multiplicity.

The second real zero from the right is x = _____, which has an _____ multiplicity.

The rightmost real zero is x = _____, which has an _____ multiplicity.

e) Select a possible function that could be represented by this graph.