Section 3.6 One-to-one Functions; Inverse Functions

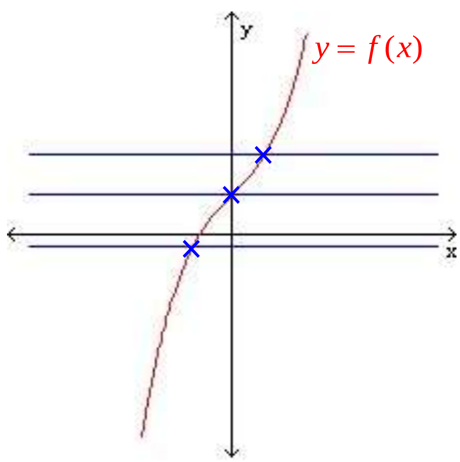
Objective 1: Understanding the Definition of a One-to-one Function

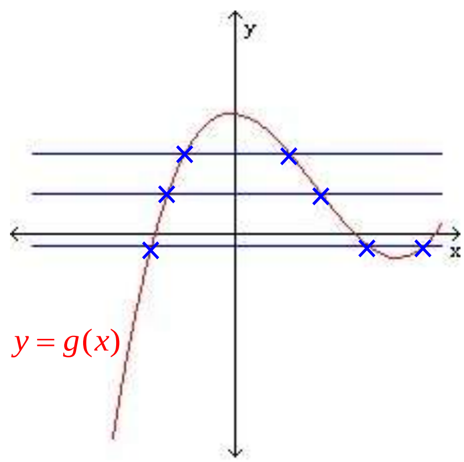
Definition **One-to-one Function**

A function f is **one-to-one** if for any values $a \neq b$ in the domain of f, $f(a) \neq f(b)$.

Interpretation: For a function *f(x)* = *y*, we know that for each *x* in the Domain there exists one and only one *y* in the Range. For a one-to-one function *f(x)* = *y*, for each *x* in the Domain there exists one and only one *y* in the Range AND for each *y* in the Range there exists one and only one *x* in the Domain.

Objective 2: Determining if a Function is One-to-one Using the Horizontal Line Test





The Horizontal Line Test If every horizontal line intersects the graph of a function f at most once, then f is one-to-one.

3.6.4 and 16
Determine whether the given functions are one-to-one.

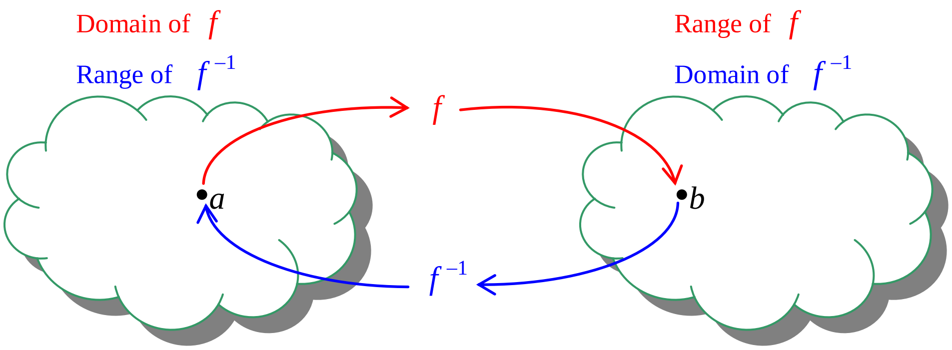
a)

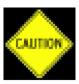
b)

Objective 3: Understanding and Verifying Inverse Functions

Every one-to-one function has an inverse function.

Definition	**Inverse Function** Let f be a one-to-one function with domain A and range B. Then f^{-1} is **the inverse function of** $\boldsymbol{f}$ with domain B and range A. Furthermore, if $f(a) = b$ then $f^{-1}(b) = a$.





Do not confuse f^{-1} with $\frac{1}{f(x)}$. The negative 1 in f^{-1} is NOT an exponent!

Inverse functions "undo" each other.

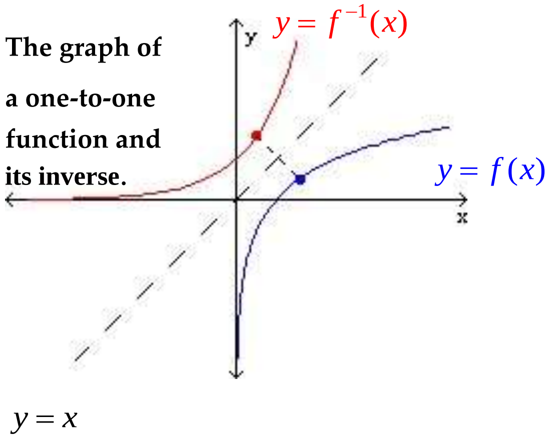
Composition Cancellation Equations $f\left(f^{-1}(x)\right) = x$ for all x in the domain of f^{-1} and $f^{-1}\left(f(x)\right) = x$ for all x in the domain of f

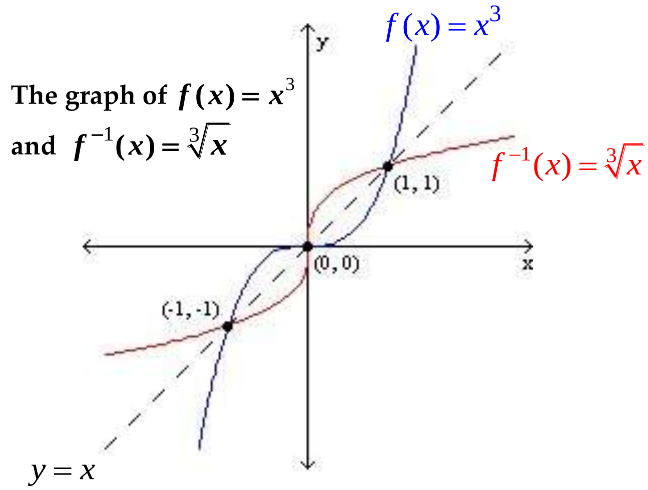
3.6.18
Determine whether *f* and *g* are inverse functions using the composition cancellation equations.

f(x) = ______________________________ *g(x)* = ______________________________

Objective 4: Sketching the Graphs of Inverse Functions

The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.
If the functions have any points in common, they must lie along the line $y = x$.





3.6.26
Sketch the graph of f^{-1} given the graph of *f*. Then state the domain and range of each using interval notation.

Domain of *f*	
Range of *f*	
Domain of f^{-1}	
Range of f^{-1}	

Objective 5: Finding the Inverse of a One-to-one Function

We know that if a point (x, y) is on the graph of a one-to-one function, then the point (y, x) is on the graph of its inverse function.

To find the inverse of a one-to-one function, replace $f(x)$ with *y*, interchange the variables x and y then solve for *y*. This is the function $f^{-1}(x)$.

3.6.34
Write an equation for the inverse function, and the state the domain and range of both given

____.

Domain of *f*	
Range of *f*	
Domain of f^{-1}	
Range of f^{-1}	

3.6.40
Write an equation for the inverse function, and the state the domain and range of both given

Domain of *f*	
Range of *f*	
Domain of f^{-1}	
Range of f^{-1}	

Inverse Function Summary

1. The inverse function f^{-1} exists if and only if the function f is one-to-one.
2. The domain of f is the same as the range of f^{-1} and the range of f is the same as the domain of f^{-1}.
3. To verify that two one-to-one functions f and *g* are inverses of each other, use the composition cancellation equations to show that $f\left(g(x)\right) = g\left(f(x)\right) = x$.
4. The graph of f^{-1} is a reflection of the graph of f about the line $y = x$. That is, for any point (a, b) that lies on the graph of *f*, the point (b, a) must lie on the graph of f^{-1}.
5. To find the inverse of a one-to-one function, replace $f(x)$ with *y*, interchange the variables x and y then solve for *y*. This is the function $f^{-1}(x)$.