

## Section 3.5 The Algebra of Functions; Composite Functions

In this section we will learn how to create a new function by combining two or more existing functions. First we will combine functions by adding, subtracting, multiplying or dividing two existing functions. The addition, subtraction, multiplication and division of functions is also known as the *algebra of functions*.

### The Algebra of Functions

Let  $f$  and  $g$  be functions. For all  $x$  such that both  $f(x)$  and  $g(x)$  are defined, the sum, difference, product, and quotient of  $f$  and  $g$  exist and are defined as follows:

1. The sum of  $f$  and  $g$ :  $(f + g)(x) = f(x) + g(x)$
2. The difference of  $f$  and  $g$ :  $(f - g)(x) = f(x) - g(x)$
3. The product of  $f$  and  $g$ :  $(fg)(x) = f(x)g(x)$
4. The quotient of  $f$  and  $g$ :  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$  for all  $g(x) \neq 0$

### Objective 1: Evaluating a Combined Function

Evaluate each function for the given value then combine using the definitions above.

### Objective 3: Finding Combined Functions and their Domains

For a number  $a$  to be in the domain of  $f + g$ ,  $a$  must be in the domain of  $f$  **and**  $a$  must be in the domain of  $g$ . In other words, the domain of  $f + g$  must be the intersection of the two domains.

(Recall that the **intersection**  $A \cap B$  of two sets of real numbers  $A$  and  $B$  is the set of all values that  $A$  and  $B$  have in common)

### The Domain of $f + g$ , $f - g$ , $fg$ and $\frac{f}{g}$

Suppose  $f$  is a function with domain  $A$  and  $g$  is a function with domain  $B$  then:

1. The domain of the sum,  $f + g$ , is the set of all  $x$  in  $A \cap B$ .
2. The domain of the difference,  $f - g$ , is the set of all  $x$  in  $A \cap B$ .
3. The domain of the product,  $fg$ , is the set of all  $x$  in  $A \cap B$ .
4. The domain of the quotient,  $\frac{f}{g}$ , is the set of all  $x$  in  $A \cap B$  such that  $g(x) \neq 0$ .

3.5.21 Find  $f+g$ ,  $f-g$ ,  $fg$ ,  $f/g$ . Determine the domain for each function.

#### Objective 4: Forming and Evaluating Composite Functions

##### Definition The Composite Function

Given functions  $f$  and  $g$ , the **composite function**,  $f \circ g$  (also called the **composition of  $f$  and  $g$** ) is defined by

$$(f \circ g)(x) = f(g(x))$$

provided  $g(x)$  is in the domain of  $f$ .



**The composition of  $f$  and  $g$  does not equal the product of  $f$  and  $g$ :  $(f \circ g)(x) \neq fg(x)$ .**

**Also, the composition of  $f$  and  $g$  does not necessarily equal the composition of  $g$  and  $f$  though this equality does exist for certain pairs of functions.**

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### Objective 5: Determining the Domain of Composite Functions

Suppose  $f$  and  $g$  are functions. For a number  $x$  to be in the domain of  $f \circ g$ ,  $x$  must be in the domain of  $g$  and  $g(x)$  must be in domain of  $f$ .

To find the domain of  $f \circ g$ :

1. Find the domain of  $g$ .
2. Exclude from the domain of  $g$  all values of  $x$  for which  $g(x)$  is not in the domain of  $f$ .

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For the given functions find the domain of  $f \circ g$  and  $g \circ f$ .

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For the given functions find the domain of  $f \circ g$  and  $g \circ f$ .