

Section 3.4 Transformations of Functions

Objectives 1-6: Using Transformations to Graph Functions

Vertical Shifts of Functions

If c is a positive real number:

The graph of $y = f(x) + c$ is obtained by shifting the graph of $y = f(x)$ vertically upward c units.

The graph of $y = f(x) - c$ is obtained by shifting the graph of $y = f(x)$ vertically downward c units.

Horizontal Shifts of Functions

If c is a positive real number:

The graph of $y = f(x + c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the left c units.

The graph of $y = f(x - c)$ is obtained by shifting the graph of $y = f(x)$ horizontally to the right c units.

Note: For $c > 0$, the graph of $y = f(x - c)$ is the graph of f shifted to the **right** c units. At first glance, it appears that the rule for horizontal shifts is the opposite of what seems natural. Substituting $x + c$ for x causes the graph of $y = f(x)$ to be shifted to the left while substituting $x - c$ for x causes the graph to shift to the right c units.

3.4.2, 3.4.13

Use the graph of a basic function and a combination of transformations to sketch the functions.

Stretches and Compressions of Functions

Suppose a is a positive real number:

The graph of $y = af(x)$ is obtained by multiplying each y -coordinate of $y = f(x)$ by a .

If $a > 1$, the graph of $y = af(x)$ is a vertical stretch of the graph of $y = f(x)$. If $0 < a < 1$, the graph of $y = af(x)$ is a vertical compression of the graph of $y = f(x)$.

3.4.43

Use the graph of a basic function and a combination of transformations to sketch the function.

Reflection of Functions about the x -Axis

The graph of $y = -f(x)$ is obtained by reflecting the graph of $y = f(x)$ about the x -axis.

Reflections of Functions about the y -Axis

The graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ about the y -axis.

When sketching a function that involves multiple transformations it is important to follow a certain “order of operations”. Below is the order in which each transformation will be performed in this text (Remember: HSRV)

- 1) Horizontal Shifts
- 2) Stretches/Compressions
- 3) Reflection about y -axis
- 4) Reflection about x -axis
- 5) Vertical Shifts

Different ordering is possible for transformations 2) through 5), but you should always perform the horizontal shift first and the vertical shift last.

3.4.25, 3.4.32, 3.4.54

Use the graph of a basic function and a combination of transformations to sketch the functions.
