Section 3.3    Graphs of Basic Functions; Piecewise Functions

Objective 1: Sketching the Graphs of the Basic Functions

We begin by discussing the graphs of two specific linear functions. Recall that a linear function has the form $f(x) = mx + b$ where $m$ is the slope of the line and $b$ represents the $y$-coordinate of the $y$-intercept.

We start our discussion of the basic functions by looking at the constant function, that is, the linear function with $m = 0$, the graph of which is a horizontal line.

1. The Constant Function $f(x) = b$

Notice that there are no arrows used at either end of the graph representing the constant function above. From this point forward in the text, unless the graph contains a definitive endpoint (shown by either an open dot or a closed dot) then it will be understood that the graph extends indefinitely in the same direction.

The identity function defined by $f(x) = x$ is another linear function with $m = 1$ and $b = 0$. It assigns to each number in the domain the exact same number in the range.

2. The Identity Function $f(x) = x$

The square function, $f(x) = x^2$, assigns to each real number in the domain the square of that number in the range.

3. The Square Function $f(x) = x^2$
The **cube function**, $f(x) = x^3$, assigns to each real number in the domain the cube of that number in the range.

4. The **Cube Function** $f(x) = x^3$

The **absolute value function**, $f(x) = |x|$, assigns to each real number in the domain the absolute value of that number in the range.

5. The **Absolute Value Function** $f(x) = |x|$

The **square root function**, $f(x) = \sqrt{x}$, is only defined for values of $x$ that are greater than or equal to zero. It assigns to each real number in the domain the square root of that number in the range.

6. The **Square Root Function** $f(x) = \sqrt{x}$

Unlike the square root function which is only defined for values of $x$ greater than or equal to zero, the **cube root function**, $f(x) = \sqrt[3]{x}$, is defined for all real numbers and assigns to each number in the domain the cube root of that number in the range.

7. The **Cube Root Function** $f(x) = \sqrt[3]{x}$
The **reciprocal function**, \( f(x) = \frac{1}{x} \), is a rational function whose domain is \( \{ x \mid x \neq 0 \} \). It assigns to each number \( a \) in the domain its reciprocal, \( \frac{1}{a} \), in the range. The reciprocal function has two asymptotes. The \( y \)-axis (the line \( x = 0 \)) is a vertical asymptote and the \( x \)-axis (the line \( y = 0 \)) is a horizontal asymptote.

8. The Reciprocal Function \( f(x) = \frac{1}{x} \).

**Objective 2: Analyzing Piecewise Defined Functions**

The absolute value function, \( f(x) = |x| \), can also be defined by a rule that has two different “pieces.”

\[
f(x) = |x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases}
\]

You can see by the graph below that the “left-hand piece” is actually a part of the line \( y = -x \) while the “right-hand piece” is a part of the line \( y = x \).

Functions defined by a rule that has more than one “piece” are called piecewise-defined functions.

3.3.14
Using the given function __________________

Find \( f(\_\_), f(\_\_), \) and \( f(\_\_) \).

Sketch the graph of the piecewise-defined function.

Determine the domain of \( f \).

Determine the range of \( f \).

3.3.18
Give the rule that describes the piecewise-defined function in the graph.