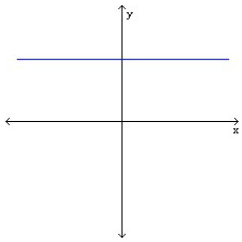
Section 3.3 Graphs of Basic Functions; Piecewise Functions

Objective 1: Sketching the Graphs of the Basic Functions

We begin by discussing the graphs of two specific linear functions. Recall that a linear function has the form $f(x) = mx + b$ where m is the slope of the line and b represents the y-coordinate of the y-intercept.

We start our discussion of the basic functions by looking at the **constant function**, that is, the linear function with $m = 0$, the graph of which is a horizontal line.

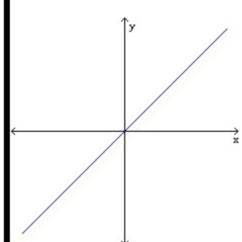
1. The Constant Function $\boldsymbol{f(x) = b}$



Notice that there are no arrows used at either end of the graph representing the constant function above. From this point forward in the text, unless the graph contains a definitive endpoint (shown by either an open dot or a closed dot) then it will be understood that the graph extends indefinitely in the same direction.

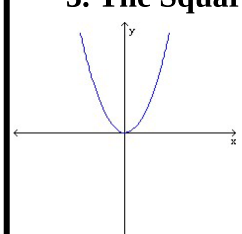
The **identity function** defined by $f(x) = x$ is another linear function with $m = 1$ and $b = 0$. It assigns to each number in the domain the exact same number in the range.

2. The Identity Function $\boldsymbol{f(x) = x}$



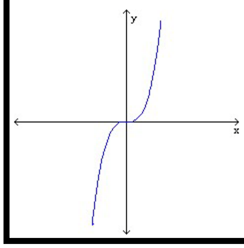
The **square function,** $f(x) = x^2$, assigns to each real number in the domain the square of that number in the range.

3. The Square Function $\boldsymbol{f(x) = x^2}$



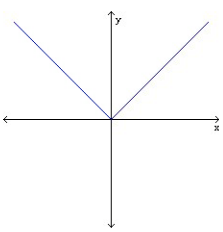
The **cube function**, $f(x) = x^3$, assigns to each real number in the domain the cube of that number in the range.

4. The Cube Function $\boldsymbol{f(x) = x^3}$



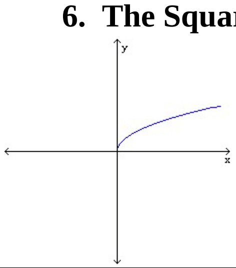
The **absolute value function**, $f(x) = |x|$, assigns to each real number in the domain the absolute value of that number in the range.

5. The Absolute Value Function $\boldsymbol{f(x) = |x|}$



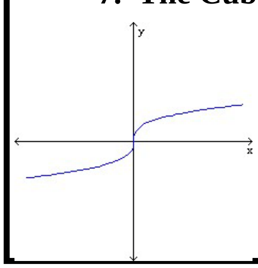
The **square root function**, $f(x) = \sqrt{x}$, is only defined for values of x that are greater than or equal to zero. It assigns to each real number in the domain the square root of that number in the range.

6. The Square Root Function $\boldsymbol{f(x) = \sqrt{x}}$



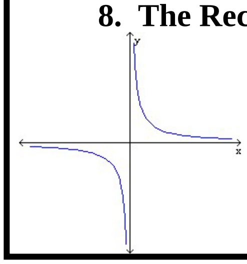
Unlike the square root function which is only defined for values of x greater than or equal to zero, the **cube root function,** $f(x) = \sqrt[3]{x}$**,** is defined for all real numbers and assigns to each number in the domain the cube root of that number in the range.

7. The Cube Root Function $\boldsymbol{f(x) = \sqrt[3]{x}}$



The **reciprocal function**, $f(x)=\dfrac{1}{x}$, is a rational function whose domain is $\{x \mid x \neq 0\}$. It assigns to each number *a* in the domain its reciprocal, $\frac{1}{a}$, in the range. The reciprocal function has two asymptotes. The *y*-axis (the line $x=0$) is a vertical asymptote and the *x*-axis (the line $y=0$) is a horizontal asymptote**.**

8. The Reciprocal Function $\boldsymbol{f(x)=\dfrac{1}{x}}$

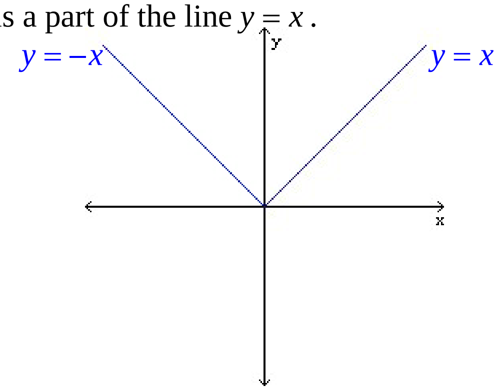


Objective 2: Analyzing Piecewise Defined Functions

The absolute value function, $f(x)=|x|$, can also be defined by a rule that has two different "pieces."

$$f(x)=|x|=\begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

You can see by the graph below that the "left-hand piece" is actually a part of the line $y=-x$ while the "right-hand piece" is a part of the line $y=x$.



Functions defined by a rule that has more than one "piece" are called piecewise-defined functions.

3.3.14
Using the given function ____________________

Find *f(___), f(______), and f(______).*

Sketch the graph of the piecewise-defined function.

Determine the domain of *f.*

Determine the range of *f.*

3.3.18
Give the rule that describes the piecewise-defined function in the graph.