

Section 3.2 Properties of a Function's Graph

Objective 1: Determining the Intercepts of a Function

An **intercept** of a function is a point on the graph of a function where the graph either crosses or touches a coordinate axis. There are two types of intercepts:

- 1) The **y-intercept**, which is the y -coordinate of the point where the graph crosses or touches the y -axis.
- 2) The **x-intercepts**, which are the x -coordinates of the points where the graph crosses or touches the x -axis.

The y-intercept:

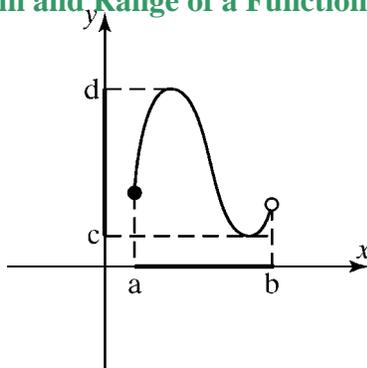
A function can have **at most** one y -intercept. The y -intercept exists if $x = 0$ is in the domain of the function. The y -intercept can be found by evaluating $f(0)$.

The x-intercept(s):

A function may have several (even infinitely many) x -intercepts. The x -intercepts, also called **real zeros**, can be found by finding all *real solutions* to the equation $f(x) = 0$. Although a function may have several zeros, only the real zeros are x -intercepts.

Objective 2: Determining the Domain and Range of a Function from its Graph

The domain is the interval $[a, b)$
while the range is the interval $[c, d]$.

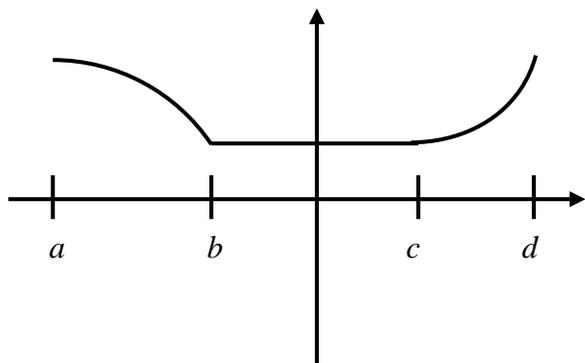


Objective 3: Determining Where a Function is Increasing, Decreasing or Constant

The graph of f rises from left to right on the interval in which f is increasing.

The graph of f falls from left to right on the interval in which f is decreasing.

A graph is constant on an open interval if the values of $f(x)$ do not change as x gets larger on the interval. In this case, the graph is a horizontal line on the interval.



The function is increasing on the interval (c, d) .

The function is decreasing on the interval (a, b) .

The function is constant on the interval (b, c) .

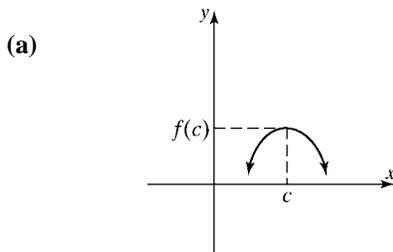
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Determine the interval(s) for which the function is (a) increasing, (b) decreasing, and (c) constant. Type your answer in interval notation. Use a comma to separate answers as needed.

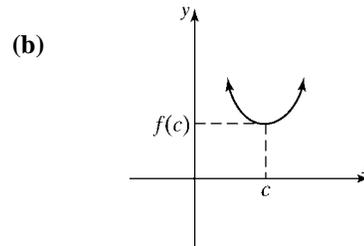
Objective 4: Determining Relative Maximum and Relative Minimum Values of a Function

When a function changes from increasing to decreasing at a point $(c, f(c))$, then f is said to have a **relative maximum** at $x = c$. The **relative maximum value** is $f(c)$.

Similarly, when a function changes from decreasing to increasing at a point $(c, f(c))$, then f is said to have a **relative minimum** at $x = c$. The **relative minimum value** is $f(c)$.



The relative maximum occurs at $x = c$, the relative maximum value is $f(c)$.



The relative minimum occurs at $x = c$, the relative minimum value is $f(c)$.

The word “relative” indicates that the function obtains a maximum or minimum value relative to some open interval. It is not necessarily the maximum (or minimum) value of the function on the entire domain.



A relative maximum cannot occur at an endpoint and must occur in an open interval. This applies to a relative minimum as well.

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Use the graph to find the following information:

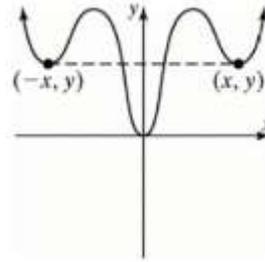
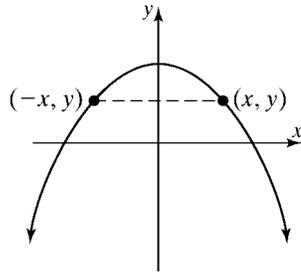
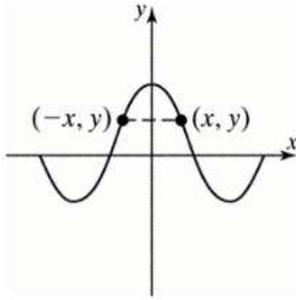
Find the number(s) for which the function obtains a relative minimum.

Find the relative minimum value.

Find the number(s) for which the function obtains a relative maximum.

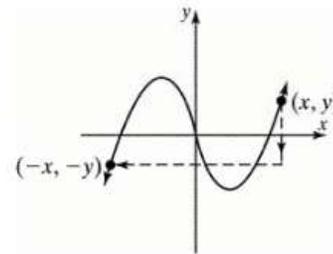
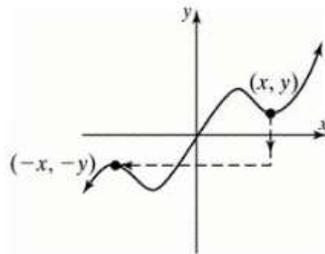
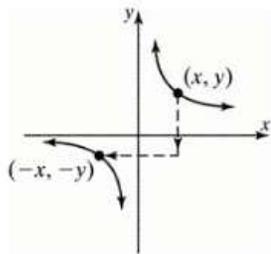
Find the relative maximum value.

Objective 5: Determining if a Function is Even, Odd or Neither



Definition Even Functions

A function f is **even** if for every x in the domain, $f(x) = f(-x)$. Even functions are symmetric about the y-axis. For each point (x, y) on the graph, the point $(-x, y)$ is also on the graph.



Definition Odd Functions

A function f is **odd** if for every x in the domain, $-f(x) = f(-x)$. Odd functions are symmetric about the origin. For each point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

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Determine if the function is even, odd, or neither.

Objective 6: Determining Information about a Function from a Graph

3.2.31

Use the graph to find the following information:

- a. What is the domain, in interval notation?
- b. What is the range, in interval notation?
- c. On what intervals is the function increasing, decreasing, or constant?
- d. For what value(s) of x does f obtain a relative minimum? What are the relative minima?
- e. For what value(s) of x does f obtain a relative maximum? What are the relative maxima?
- f. What are the real zeros of f ?
- g. What is the y -intercept?
- h. For what values of x is $f(x)$ less than or equal to 0?
- i. For how many values of x is $f(x) = \underline{\hspace{2cm}}$?
- j. What is the value of $f(\underline{\hspace{2cm}})$?