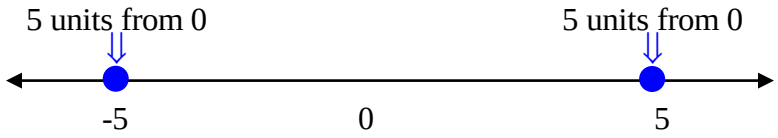
Section 1.8 Absolute Value Equations and Inequalities

Objective 1: Solving an Absolute Value Equation

The absolute value of a number *x*, written as $|x|$, represents the **distance** from a number *x* to 0 on the number line. Consider the equation $|x| = 5$. To solve for *x*, we must find all values of *x* that are 5 units away from 0 on the number line. The two numbers that are 5 units away from 0 on the number line are $x = -5$ and $x = 5$ as shown in Figure 8.



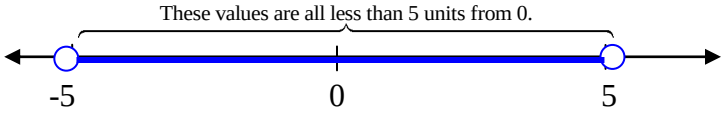
If $|x| = 5$, then $x = -5$ or $x = 5$. The solution set is $\{-5, 5\}$.

1.8.2

Solve the equation for *x*. Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.

Objective 2: Solving an Absolute Value "Less Than" Inequality

The solution to the inequality $|x| < 5$ consists of all values of *x* whose distance from 0 is less than 5 units on the number line.



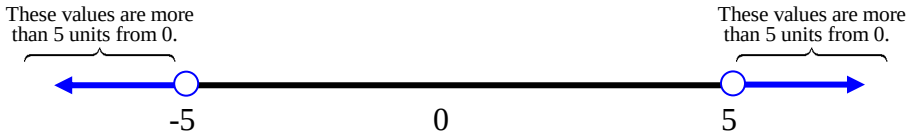
If $|x| < 5$, then $-5 < x < 5$. The solution set is $\{x \mid -5 < x < 5\}$ in set builder notation or $(-5, 5)$ in interval notation.

1.8.8

Solve the inequality. Express the solution using interval notation. Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.

Objective 3: Solving an Absolute Value "Greater Than" Inequality

For the solution to the inequality $|x| > 5$, notice that we are now looking for all values of x that are more than 5 units away from 0. The solution is the set of all values of x greater than 5 combined with the set of all values of x less than -5.



If $|x| > 5$, then $x < -5$ or $x > 5$. The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$ in set builder notation or $(-\infty, -5) \cup (5, \infty)$ in interval notation.

When solving an absolute value equation or inequality, it is necessary to first rewrite it in standard form.

1.8.15
Solve the inequality. Express the solution using interval notation. Simplify your answer. Type an integer or a fraction. Use a comma to separate answers as needed.

$|5x+1| > 3$ is NOT equivalent to $-3 > 5x+1 > 3$. In addition, a common error on this type of problem is to write $5x+1 > -3$ for the first inequality instead of $5x+1 < -3$. Think carefully about the meaning of the inequality before writing it.

MOST IMPORTANT CONCLUSION. KNOW THIS!!!!!

ABSOLUTE VALUE EQUATIONS AND INEQUALITY PROPERTIES

Let u be an algebraic expression and let c be a real number such that $c > 0$, then:

1. $|u| = c$ is equivalent to $u = -c$ or $u = c$.
2. $|u| < c$ is equivalent to $-c < u < c$.
3. $|u| > c$ is equivalent to $u < -c$ or $u > c$.