

Section 1.4 Quadratic Equations

In section 1.1 we studied linear equations of the form $ax + b = c$, $a \neq 0$. These equations are also known as 1st order polynomial equations. In this section, we will learn how to solve 2nd order polynomial equations. Second order polynomial equations are called **quadratic equations**.

Definition Quadratic Equation in One Variable

A **Quadratic Equation in One Variable** is an equation that can be written in the form $ax^2 + bx + c = 0$, $a \neq 0$. Quadratic equations in this form are said to be in *standard form*.

Objective 1: Solving Quadratic Equations by Factoring and the Zero Product Property

Some quadratic equations can be easily **solved by factoring** and by using the following important property.

Property The Zero Product Property

If $AB = 0$ then $A = 0$ or $B = 0$.

The zero product property says that if two factors multiplied together are equal to zero, then at least one of the factors must be zero.

1.4.5

Solve the equation by factoring. Use a comma to separate answers as needed. Simplify your answer.

Objective 2: Solving Quadratic Equations using the Square Root Property

Any quadratic equation of the form $x^2 - c = 0$ where $c > 0$ can be solved by factoring the left side as $(x - \sqrt{c})(x + \sqrt{c}) = 0$ thus the solutions are $x = \pm\sqrt{c}$. Quadratic equations of this form can be more readily solved by using the following **square root property**.

Property The Square Root Property

The solution to the quadratic equation $x^2 - c = 0$ or equivalently $x^2 = c$ is $x = \pm\sqrt{c}$.

1.4.9

Solve by using the square root property.

Objective 3: Solving Quadratic Equations by Completing the Square

Every quadratic equation can be written in the form $(x-h)^2 = k$ using a method known as **completing the square**. Consider the following perfect square trinomials:

$$x^2 + 2x + 1 = (x+1)^2$$

$\begin{array}{c} \text{└──┬──┘} \\ \uparrow \\ (\frac{1}{2} * 2)^2 = 1 \end{array}$

$$x^2 - 6x + 9 = (x-3)^2$$

$\begin{array}{c} \text{└──┬──┘} \\ \uparrow \\ (\frac{1}{2} * (-6))^2 = 9 \end{array}$

$$x^2 - 7x + \frac{49}{4} = (x - \frac{7}{2})^2$$

$\begin{array}{c} \text{└──┬──┘} \\ \uparrow \\ (\frac{1}{2} * (-7))^2 = \frac{49}{4} \end{array}$

In each perfect square trinomial above, notice the relationship between the coefficient of the linear term (x -term) and the constant term. The constant term of a perfect square trinomial is equal to the square of $\frac{1}{2}$ of the linear coefficient.

To solve a quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ by completing the square, follow these steps:

Steps for Solving $ax^2 + bx + c = 0$, $a \neq 0$ by Completing the Square
<ol style="list-style-type: none">1. If $a \neq 1$, divide the equation by a.2. Move all constants to the right-hand side.3. Take half the coefficient of the x term, square it, and add it to both sides of the equation.4. The left-hand side is now a perfect square. Rewrite it as a binomial squared.5. Use the square root property to solve for x.

1.4.17 Solve the quadratic equation by completing the square.

Objective 4: Solving Quadratic Equations Using the Quadratic Formula

The **quadratic formula** can be used to solve any quadratic equation.

The Quadratic Formula

The solution to the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Solve the equation using the quadratic formula. Simplify your answer. Type exact answers, using radicals as needed. Use a comma to separate answers.

Objective 5: Using the Discriminant to Determine the Type of Solutions of a Quadratic Equation

Given a quadratic equation of the form $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is called the **discriminant**. Knowing the value of the discriminant can help us determine the number and nature of the solutions to a quadratic equation.

The Discriminant

Given a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, the expression $D = b^2 - 4ac$ is called the **discriminant**.

If $D > 0$, then the quadratic equation has two real solutions.

If $D < 0$, then the quadratic equation has two non-real solutions.

If $D = 0$, then the quadratic equation has exactly one real solution.

1.4.29 Use the discriminant to determine the number and nature of the solutions of the quadratic equation.