Section 5.2: Factoring Trinomials

Objective 1: Factor Trinomials with a Leading Coefficient Equal to One \((ax^2 + bx + c, \ a = 1)\)

The binomial product \((x+a)(x+b)\) is said to be a factored form of the trinomial \(x^2 + (a+b)x + ab\). Our goal is to start with a trinomial and try to find two binomials whose product is equal to the given trinomial. If we carefully look at last two terms of the trinomial \(x^2 + (a+b)x + ab\) we see that the middle term, \((a+b)x\), is the sum of the products of the inside and outside terms of the two binomials. The third term, \(ab\), is the product of the last two terms of each binomial. Thus, when factoring trinomials of the form \(x^2 + bx + c\), we must find two numbers whose product is \(c\) and whose sum is \(b\).

Note: A polynomial is a prime polynomial if it cannot be factored by the methods of this section.

5.2.9 Factor the trinomial or state that the trinomial is prime.

5.2.13 Factor completely

5.2.19 Factor completely

Objective 2: Factor Trinomials with a Leading Coefficient Not Equal to One \((ax^2 + bx + c, \ a \neq 1)\)

We can try to factor trinomials of the form \(ax^2 + bx + c\) where \(a \neq 1\) using a trial and error method. That is, we basically “guess” until we stumble upon the correct two binomial factors. For example, suppose that we wish to factor the trinomial \(6x^2 - 7x - 3\). The possible first terms of the two binomial factors are \(x\) and \(6x\) or \(2x\) and \(3x\). The only integer factors of the constant, \(-3\), are \((1)\cdot(-3)\) and \((3)\cdot(-1)\). This means that there are 8 possible combinations of potential pairs of binomial factors for this trinomial. The possibilities are:

1. \((x-1)(6x + 3)\)
2. \((x+1)(6x - 3)\)
3. \((6x-1)(x + 3)\)
4. \((6x+1)(x-3)\)
5. \((2x-1)(3x+3)\)
6. \((2x+1)(3x-3)\)
7. \((3x-1)(2x+3)\)
8. \((3x+1)(2x-3)\)

If we use the FOIL method to multiply each of the 8 possibilities, we see that the final two binomial factors on the list is the correct factorization because \((3x+1)(2x-3) = 6x^2 - 7x - 3\).
5.2.21 Use trial-and-error to factor the trinomial, or state that the trinomial is prime.

5.2.24 Use trial-and-error to factor the trinomial, or state that the trinomial is prime.

Although this “guessing” method works, it can often be a long and arduous process especially when the leading coefficient and constant coefficient have several factors. Fortunately, there is another more methodical approach to factoring trinomials.

We now outline this four step method using the same trinomial $6x^2 - 7x - 3$ as our example.

**ac Method of Factoring Trinomials of the form $ax^2 + bx + c$, $a \neq 0, a \neq 1$**

Example: $6x^2 - 7x - 3$, $a = 6$, $b = -7$, $c = -3$

**Step 1** Multiply $a \cdot c$

$(6) \cdot (-3) = -18$

**Step 2** Find two numbers whose product is $ac$ and whose sum is $b$.

The two numbers are $-9$ and $2$ because $(-9) \cdot (2) = -18$ and $(-9) + (2) = -7$.

**Step 3** Rewrite the middle term as two terms using the numbers found in step 2.

$6x^2 - 9x + 2x - 3$

$= 6x^2 - 9x + 2x + 3$

$= (2x - 3)(3x + 1)$

5.2.27 Use the ac method to factor the trinomial, or state that the trinomial is prime.
Objective 4: Factor trinomials using substitution

Using substitution, we can sometimes factor more complicated trinomials. Consider the trinomial $x^{10} - 7x^5 + 12$. Notice that the degree of the leading term is twice the degree of the middle term.

\[
\begin{align*}
\text{Degree of leading term} & = 2 \\
\text{Degree of middle term} & = 5
\end{align*}
\]

$\quad x^{10} - 7x^5 + 12$

Trinomials like this are called quadratic in form because the variable part of the first term can be written as the square of the variable factor in the middle term.

\[
x^{10} - 7x^5 + 12 = (x^5)^2 - 7(x^5) + 12
\]

If we substitute a new variable, say $u$, for the variable factor in the middle term, the result is a second-degree trinomial.

**Substitute $u$ for $x^5$:** $\quad (x^5)^2 - 7(x^5) + 12 = u^2 - 7u + 12$

We then factor this trinomial, if possible.

**Factor:** \quad $u^2 - 7u + 12 = (u - 4)(u - 3)$

Finally, we write the factored form in terms of the original variable.

**Substitute $x^5$ for $u$:** \quad $(u - 4)(u - 3) = (x^5 - 4)(x^5 - 3)$

So, $\quad x^{10} - 7x^5 + 12 = (x^5 - 4)(x^5 - 3)$.

5.2.41 Use substitution to factor the trinomial.

5.2.44 Use substitution to factor the trinomial.