MATH 220	NAME ______________________
FINAL EXAM	STUDENT NUMBER ______________________
Fall 2012	INSTRUCTOR ______________________
VERSION A	SECTION NUMBER ______________________

On your scantron, write and bubble your PSU ID, Section Number, and Test Version. Failure to correctly code these items may result in a loss of 5 points on the exam.

On your scantron, bubble letters corresponding to your answers on indicated questions. It is a good idea for future review to circle your answers in the test booklet.

Check that your exam contains 30 multiple-choice questions, numbered sequentially.

Answer Questions 1–30 on your scantron.

Each question is worth 5 points.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED DURING THIS EXAMINATION.

THE USE OF NOTES OF ANY KIND IS NOT PERMITTED DURING THIS EXAMINATION.

1. Which of the following matrices are in **reduced echelon form**?

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

a) A and B only

b) A and C only

c) B and C only

d) C only

2. For which value(s) of h is $\mathbf{b}$ in Span$\{\mathbf{a_1}, \mathbf{a_2}\}$?

$$\mathbf{a_1} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \mathbf{a_2} = \begin{bmatrix} -3 \\ -7 \\ 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -3 \\ 3 \\ h \end{bmatrix}$$

a) $h = 6$

b) $h \neq 6$

c) $h = 3$

d) $h \neq 3$

3. How many solutions does the homogeneous equation $A\mathbf{x} = \mathbf{0}$ have?

$$A = \begin{bmatrix} 2 & 5 & -3 & 4 \\ 0 & 1 & 9 & 2 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

a) No solutions

b) Exactly one solution

c) Exactly two solutions

d) Infinitely many solutions

4. Which of the following sets are **linearly independent**?

a) $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ -7 \\ 15 \end{bmatrix}, \begin{bmatrix} -1 \\ 8 \\ 4 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 5 \\ 8 \\ -6 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 4 \\ 1 \end{bmatrix} \right\}$

5. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ with $T\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)=\begin{bmatrix}2\\5\end{bmatrix}$ and $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right)=\begin{bmatrix}7\\-2\end{bmatrix}$.

What is $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)$?

a) $\begin{bmatrix}5\\-7\end{bmatrix}$

b) $\begin{bmatrix}16\\1\end{bmatrix}$

c) $\begin{bmatrix}-2\\7\end{bmatrix}$

d) $\begin{bmatrix}9\\3\end{bmatrix}$

6. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ given by

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right)=\begin{bmatrix}3x_1-5x_2+x_3\\x_1-6x_3\end{bmatrix}.$$

What is the standard matrix for T?

a) $\begin{bmatrix}3 & 1\\-5 & -6\\1 & 0\end{bmatrix}$

b) $\begin{bmatrix}3 & -5 & 1\\1 & 0 & -6\end{bmatrix}$

c) $\begin{bmatrix}3 & 1\\-5 & 0\\1 & -6\end{bmatrix}$

d) $\begin{bmatrix}3 & -5 & 1\\1 & -6 & 0\end{bmatrix}$

7. Find the inverse of $A = \begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$.

a) $\begin{bmatrix} 3 & -8 \\ -2 & 5 \end{bmatrix}$

b) $\begin{bmatrix} -3 & 8 \\ 2 & -5 \end{bmatrix}$

c) $\begin{bmatrix} -5 & 2 \\ 8 & -3 \end{bmatrix}$

d) $\begin{bmatrix} 5 & -2 \\ -8 & 3 \end{bmatrix}$

8. Which of the following is **NOT** in Nul A?

$$A = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 1 & 2 & 3 & 5 \\ 2 & 0 & 2 & -2 \end{bmatrix}$$

a) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

9. Let
$$A = \begin{bmatrix} 1 & -1 & 2 & 2 \\ 0 & 0 & 4 & 6 \\ 1 & 4 & 2 & 1 \end{bmatrix}.$$
Determine the dimension of Col A and the dimension of Nul A.

a) Dimension of Col $A = 2$; Dimension of Nul $A = 2$

b) Dimension of Col $A = 2$; Dimension of Nul $A = 1$

c) Dimension of Col $A = 1$; Dimension of Nul $A = 3$

d) Dimension of Col $A = 3$; Dimension of Nul $A = 1$

10. Let A be an $n \times n$ matrix. Which of the following must be **true** if A is invertible?

a) The dimension of Col $A = n$.

b) det $A = 0$

c) The dimension of Col A equals the dimension of Nul A.

d) The columns of A are linearly dependent.

11. Compute $\det A$ given

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ -1 & -4 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \end{bmatrix}.$$

a) 0

b) 24

c) 72

d) -72

12. Assume $\det A = 6$ where

$$A = \begin{bmatrix} 0 & 1 & 3 \\ 4 & 5 & 7 \\ a & b & c \end{bmatrix}.$$

Find the determinant of the matrix

$$\begin{bmatrix} 0 & -1 & -3 \\ a & b & c \\ 8 & 10 & 14 \end{bmatrix}.$$

a) 3

b) -3

c) 12

d) -12

13. Find $\det A$ given

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 3 \\ 1 & 1 & 0 & 1 \\ 2 & 3 & 1 & 0 \end{bmatrix}.$$

a) -8

b) 0

c) 2

d) 6

14. Which of the following is an eigenvector of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 3 & 0 \\ -1 & 1 & 4 \end{bmatrix}$$

corresponding to the eigenvalue $\lambda = 3$?

a) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 5 \\ -5 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ 6 \\ 6 \end{bmatrix}$

d) $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$

15. Which of the following is an eigenvalue of the matrix A^2 where

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 4 & 5 & 0 \\ 0 & 8 & -2 \end{bmatrix}?$$

a) 2

b) 10

c) 1

d) -4

16. Find a basis for the eigenspace of

$$A = \begin{bmatrix} 4 & 3 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

corresponding to the eigenvalue $\lambda = 4$.

a) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

17. Which of the following statements about the matrices A and B is true?

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

a) 0 is an eigenvalue of A and 5 is an eigenvalue of B.

b) 4 is an eigenvalue of A and -5 is an eigenvalue of B.

c) 1 is an eigenvalue of A and 5 is an eigenvalue of B.

d) 4 is an eigenvalue of A and 2 is an eigenvalue of B.

18. Let A be a 3×3 matrix with eigenvalues 1 and 3. Let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for the eigenspace corresponding to 1, and let $\{\mathbf{v}_3\}$ be a basis for the eigenspace corresponding to 3, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and } \mathbf{v}_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}.$$

Which of the following statements is true?

a) A diagonalization of A is given by $P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

b) A diagonalization of A is given by $P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

c) A diagonalization of A is given by $P = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

d) A is not diagonalizable.

19. Let

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}.$$

Find a diagonal matrix D such that $A = PDP^{-1}$ for some matrix P.

a) $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

c) $\begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$

d) The matrix A is not diagonalizable.

20. Which of the following is a unit vector pointing in the same direction as

$$\begin{bmatrix} 1 \\ 0 \\ -2 \\ 2 \end{bmatrix}?$$

a) $\begin{bmatrix} 1/9 \\ 0 \\ -2/9 \\ 2/9 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 1/3 \\ 0 \\ -2/3 \\ 2/3 \end{bmatrix}$

21. Which of the following is true about the sets of vectors S and T?

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} \right\}, \qquad T = \left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -9 \\ 3 \end{bmatrix} \right\}$$

a) The set S is orthogonal, and the set T is not orthogonal.

b) The set S is not orthogonal, and the set T is orthogonal.

c) The sets S and T are both orthogonal.

d) Neither of the sets S and T is orthogonal.

22. Let $\mathbf{y} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}$, and let $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Which of the following represents the projection of the vector $\mathbf{y}$ onto the vector $\mathbf{u}$?

a) $\begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$

b) $\begin{bmatrix} 1/2 \\ -1/2 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$

d) $\begin{bmatrix} 4/3 \\ -4/3 \\ 2/3 \end{bmatrix}$

23. Which of the following represents the distance from the vector $\mathbf{y}$ to the line through the vector $\mathbf{u}$ and the origin, where

$$\mathbf{y} = \begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix}, \text{ and } \mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}?$$

a) 27

b) $\sqrt{27}$

c) 8

d) $\sqrt{8}$

24. Find $\text{Proj}_W\mathbf{y}$, the orthogonal projection of $\mathbf{y}$ onto W, where $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $W = \text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\}$.

a) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1/2 \\ 1/2 \end{bmatrix}$

d) $\begin{bmatrix} 7/2 \\ 1 \\ 1 \end{bmatrix}$

25. Find the closest point to $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \end{bmatrix}$ in the subspace $H = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix} \right\}$.

a) $\begin{bmatrix} 1 \\ -1/3 \\ 5/3 \\ 5/3 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1/3 \\ -5/3 \\ -5/3 \end{bmatrix}$

d) $\begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \end{bmatrix}$

26. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$. Which of the following is an orthogonal basis for W?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

27. Let $\left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \right\}$ be an orthogonal basis for a subspace H. Find an orthonormal basis for H.

a) $\left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 4/5 \\ -3/5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 4/25 \\ -3/25 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \right\}$

28. Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Find the **orthogonal** matrix P so that $A = PDP^T$ where D is a diagonal matrix.

a) $P = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

b) $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

c) $P = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

d) $P = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

29. For which of the following matrices is there an orthonormal basis of eigenvectors for $\mathbb{R}^3$?

a) $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 7 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & -1 \\ 3 & -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

30. Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$. Find a diagonal matrix D so that $A = PDP^T$ where P is an orthogonal matrix.

a) $D = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$

b) $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$

c) $D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

d) D does not exist.

Answers

1. C
2. C
3. D
4. B
5. A
6. B
7. C
8. D
9. D
10. A
11. D
12. C
13. A
14. B
15. C
16. C
17. C
18. A
19. D
20. D
21. B
22. C
23. D
24. C
25. A
26. D
27. B
28. A
29. B
30. A