Section 2.5  Linear Equations in Two Variables

Objective 1: Find the Slope of a Line

In mathematics, the steepness of a line can be measured by computing the line’s slope. Every non-vertical line has slope. Vertical lines are said to have no slope.

A line going up from left to right has positive slope, a line going down from left to right has negative slope, while a horizontal line has zero slope. We use the variable $m$ to describe slope.

\[ \text{Slope} = m \]

The slope can be computed by comparing the vertical change (the rise) to the horizontal change (the run). Given any two points on the line, the slope $m$ can be computed by taking the quotient of the rise over the run.

**Definition of Slope**

If $x_1 \neq x_2$, the slope of a line passing through distinct points $(x_1, y_1)$ and $(x_2, y_2)$ is

\[
m = \frac{\text{rise}}{\text{run}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

2.5.5 Find the slope of the line containing the two given points

**Objective 2: Graph a Line Using the Slope and a Point**

If we know a point on a line and the slope, we can quickly sketch the line.

2.5.10 Find the slope and the $y$-intercept and use them to graph the linear function.
2.5.11 Graph the line that contains the point ( , ) and has a slope of __________.

Objective 3: Determine the Relationship Between Two Lines

Parallel lines have the same slope but different y-intercepts.
Perpendicular lines intersect each other at right angles (90°)
    The product of the slopes of perpendicular lines is \(-1\).
    The slopes of perpendicular lines are negative reciprocals of each other.

2.5.13 Determine whether the two lines are parallel, perpendicular, coinciding, or only intersecting.

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Objective 4: Write the equation of a line from given information

Given the slope \( m \) of a line and a point on the line, \((x_1, y_1)\), we can use what is known as the point-slope form of the equation to determine the equation of the line.

**The Point-Slope Form of the Equation of a Line**

Given the slope of a line \( m \) and a point on the line \((x_1, y_1)\), the point-slope form of the equation of a line is given by \( y - y_1 = m(x - x_1) \).

**The Slope-Intercept Form of the Equation of a Line**

Given the slope of a line \( m \), and the y-intercept \( b \), the slope-intercept form of the equation of a line is given by \( y = mx + b \).
The Standard Form Equation of a Line
The standard form of an equation of a line is given by $Ax + By = C$ where $A, B,$ and $C$ are integers (not fractions) and $A \geq 0$.

2.5.24 Find an equation of the line having the given slope and containing the given point. Write the equation in slope-intercept form.

2.5.28 Find the equation of the line that contains the given points. Write the equation in slope-intercept form.

SUMMARY OF FORMS OF EQUATIONS OF LINES

1. $y - y_1 = m(x - x_1)$  
   **Point-Slope Form**
   Slope is $m$ and $(x_1, y_1)$ is a point on the line.

2. $y = mx + b$  
   **Slope-Intercept Form**
   Slope is $m$ and $y$-intercept is at $(0, b)$.

3. $Ax + By = C$  
   **Standard Form**
   $A, B$ and $C$ are real numbers with $A$ and $B$ both not $0$ with $A \geq 0$.

4. $y = b$  
   **Horizontal Line**
   Slope is $0$, $y$-intercept is at $(0, b)$.

5. $x = a$  
   **Vertical Line**
   Undefined slope, $x$-intercept is at $(a, 0)$. 
Objective 5: Write equations of parallel and perpendicular lines

**Parallel lines** have the same slope but different y-intercepts. **Perpendicular lines** intersect each other at right angles (90°)
- The product of the slopes of perpendicular lines is $-1$.
- The slopes of perpendicular lines are negative reciprocals of each other.

2.5.33 Write an equation of the line parallel to the given line that passes through the given point. Write your answer in slope-intercept form.

2.5.35 Write an equation of the line perpendicular to the given line that passes through the given point. Write your answer in slope-intercept form.